ॐ

OM
5

Sujal Patel ([redacted])
(9574234622).

ECE

PM 1 (B).

ACE ACADEMY

CONTROL SYSTEMS

☆ CONTROL SYSTEMS:

⇒ **Books:**

Gate ① Control System → NISE

IES ② Control system → Nagrath & Gopal.

③ Automatic Control System → B.C. Kuo.

IES ④ Control System : Principal & Design → M Gopal.

⑤ Modern CS → Ogata.

* Topics:

→ TF, BD, SFG → 1M (or) 2M

→ TDA → Transient Analysis, Steady State Analysis → 2M

→ Stability Techniques → Time Domain tech. ⇒ RH | RL ; Freq. Domain tech. ⇒ BP | NP → 4M

→ Compensator | controllers.

→ State Space Analysis. → 2M

☆ Introduction:

* Transfer, Block Diagram & SFG:

⇒ Transfer function is a Mathematical equivalent model for the system.

⇒ $TF = \underbrace{\frac{1}{S+1}}$

⇓

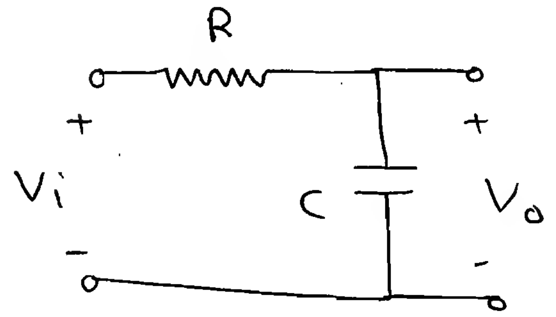
No. of Storage elements

(or)

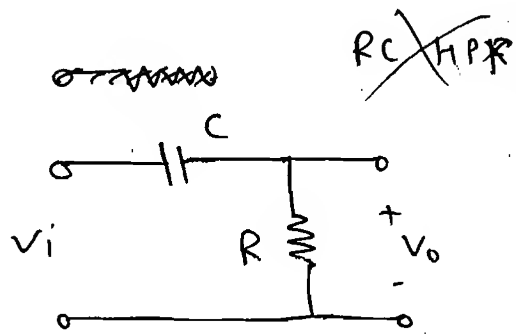
No. of Time constant.

→ Objective of the CS : Set desired / Accurate output.

⇒

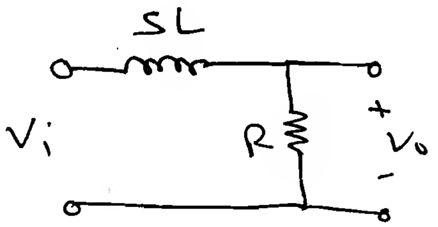


RC LPF ✓



→ $\frac{V_o(s)}{V_i(s)} = \frac{1/sC}{R + 1/sC}$

$\frac{V_o(s)}{V_i(s)} = \frac{1}{sCR+1}$.



L-R LPF ✓

Let, $\tau = RC$.

$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{s\tau+1} \Rightarrow \frac{C(s)}{R(s)} = \left(\frac{1}{s+1}\right)$.

$\therefore \tau = \underbrace{R}_{1M}\underbrace{C}_{1\mu F} = 1$

i.e $\frac{C(s)}{R(s)} = \frac{1}{s+1}$ for LPF.

⇒ Why LPF?

Noise → Eliminated → (LPF)

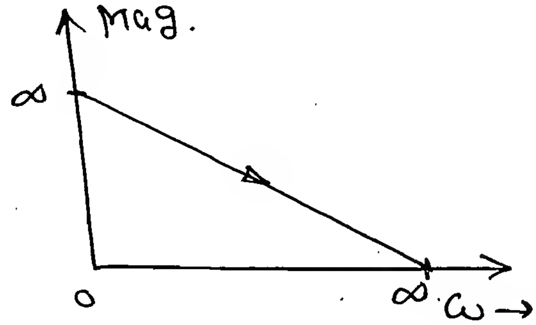
⇓

High freq. → Eliminated

① Noise get eliminated by LPF.

② Components are more stable at LF.

⇒ System :
$$= \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^n(1+s\tau_a)(1+s\tau_b)\cdots}$$



⇒ **Always selecting** :

⊙ Poles > Zero ⇒ For LPF

⇒ Then only it is Strictly Proper TF

⊙ When Poles = zeros

⇒ it act as {LP, HP, BP, BS} → Proper TF.

→ zero at origin is not acceptable.

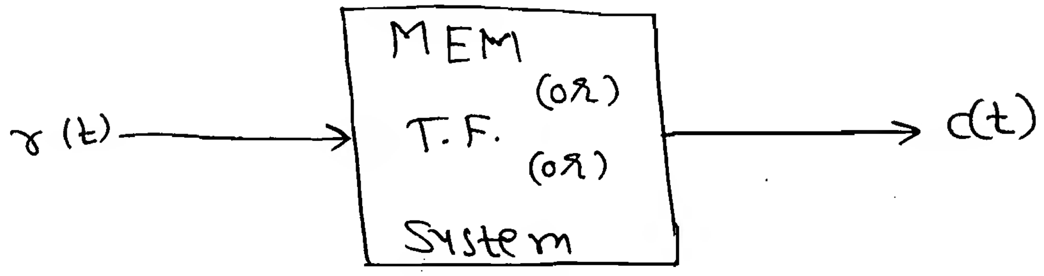
⊙ when Poles < zeros ⇒ Improper T.F. ⇒ HPF.

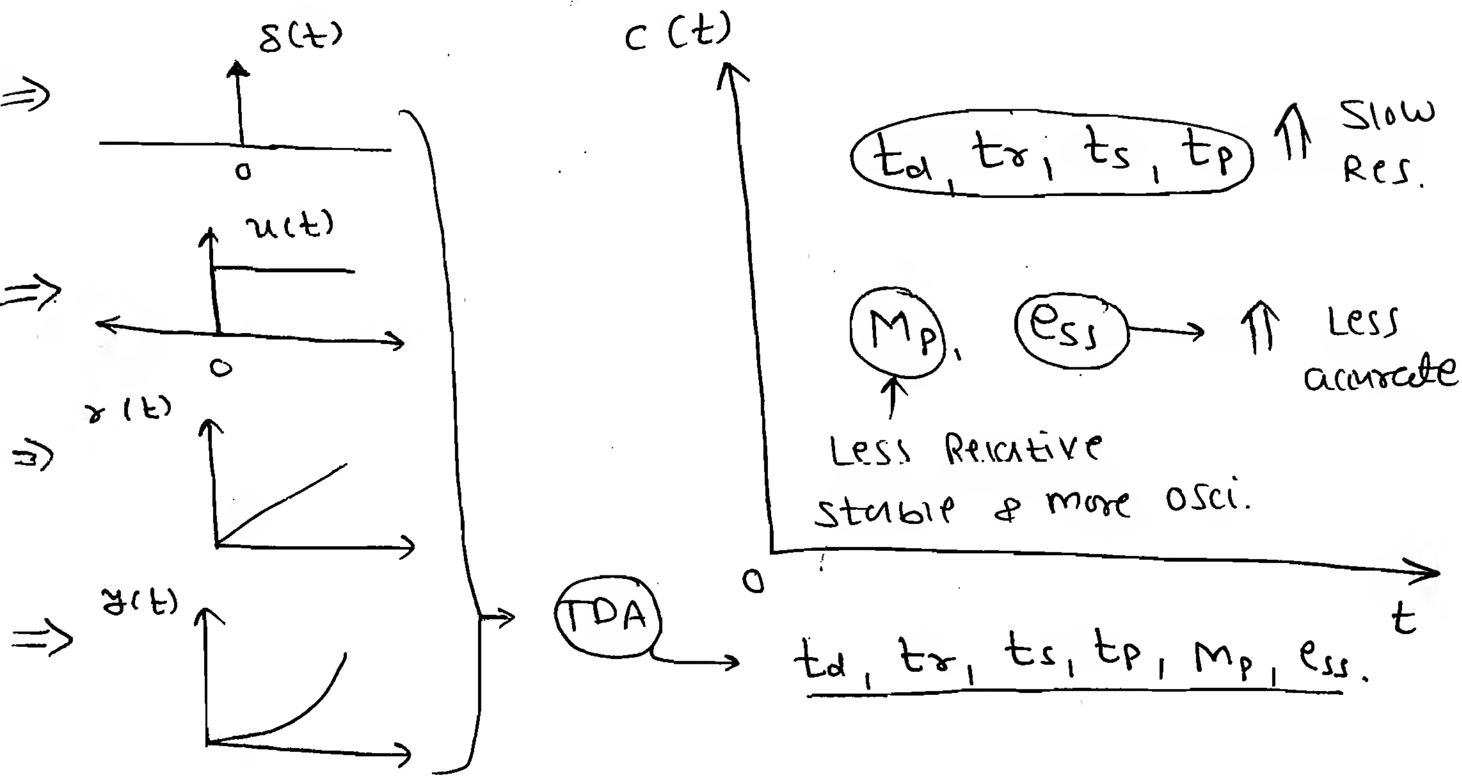
⇒ BD, SFG ⇒ To find the overall TF of the system.

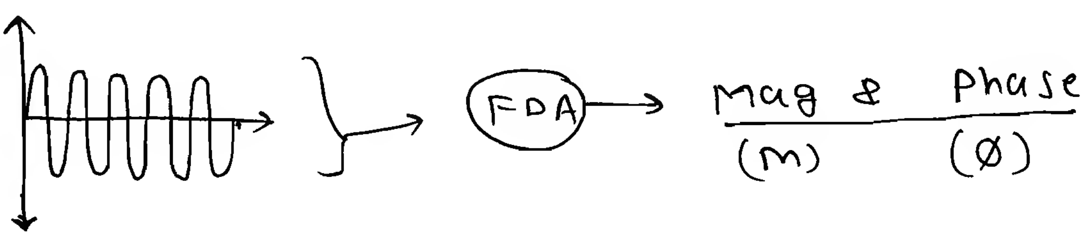
* Time Domain Analysis:

⇒ The objective of the TDA is used to evaluate the performance of the system w.r.t. time.

⇒







⇒ Freq. domain analysis is used to find GM & PM.

* Control System Specification:

⇒ Speed → t_r, t_s ↓↓ → Quick Res.

⇒ Accuracy → e_{ss} ↓ → Small → More Accurate.

⇒ Stability ⇒ GM & PM

- ↑↑
  - More R.S. (Adv.)
  - Slow Res. (dis.)
- ↓↓
  - Less R.S. (dis.adv.)
  - More oscillatory (dis.adv.)
  - ↓
  - $t_r \downarrow$, $t_s \uparrow$

→ Optimum

Value of GM ⇒ 5 dB to 10 dB

PM ⇒ 30° to 40°.

→ TDA should be insensitive w.r.t. to unwanted parameters such as Temperature, Noise & Disturbance.

⇒ e_{ss} : Steady State error.

⇒ M_p : Peak overshoot.

⇒ t_d : Delay time.

⇒ t_r : Rise time.

⇒ t_p : Propogation time.

⇒ t_s : Setting time.

* Stability (for Closed loop system).

OL System → $G(s)$ (OLTF of a system)

OL System and Closed loop system → Always defined interms of OLTF.

Closed loop system → $G(s)\,[H(s)=1]$ = OLTF of a unity FB sys.

$$CLTF = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

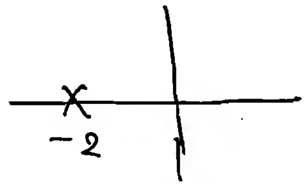
Closed loop system → $G(s)\cdot H(s)$ = OLTF of a Non-unity FB sys.

$$CLTF = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}$$

$\boxed{Q}$ OLTF of a unity FB sys is $G(s) = \frac{10}{s-8}$. Then system is ______.

Ans:

$$CLTF = \frac{G}{1+G} = \frac{10}{s+2}$$

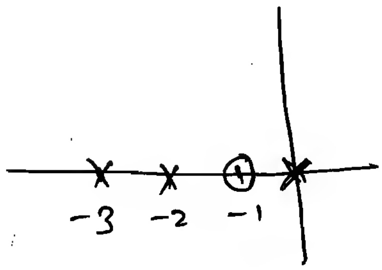


So, $\boxed{\text{Stable}}$.

* Why there is no need of stability technique for OL system?

⇒ OLTF of a system is,

$$G(s) = \frac{s+1}{s^2(s+2)(s+3)}$$



→ Poles and zeros location are identified directly from $G(s)$.

⇒ CL SYS.:

OLTF of a unity FB system is,

$$G(s) = \frac{s+1}{s^2(s+2)(s+3)}$$

$$CLTF = \frac{G}{1+G} = \frac{s+1}{s^4 + 5s^3 + 6s^2 + s + 1}$$

⇒ The Feedback changes the locations of the poles. Identification of new location of the poles are very difficult, Hence we need a stability technique for closed loop stability.

⇒ Stability Technique:

Priority

1. Nyquist → No. of poles on RL, Range of K, GM & PM can be find.
2. RL → Nature of the system.
3. BP → GM & PM.
4. RH.

→ The T-D technique gives the transient Analysis and steady state (SS) Analysis.

⇒ The F-D technique gives the only steady state (SS) Analysis.

⇒ The stability Analysis is a steady state Analysis.

* Transportation Delay | Lag System.

$\Rightarrow \quad L[g(t-\tau)] = e^{-s\tau} G(s).$

$\uparrow$ delay (under τ)

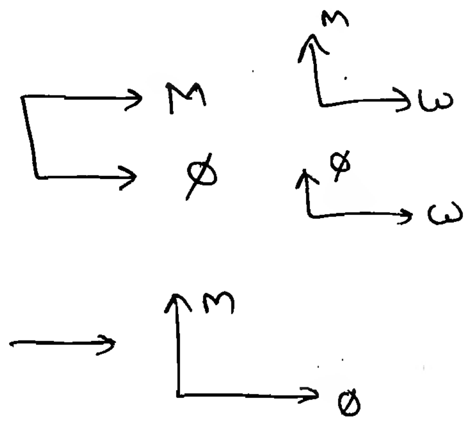
TDA: $e^{-s\tau} \Rightarrow (1-s\tau) + \frac{(s\tau)^2}{2!} + \dots + \infty$. → Neglected infinite poles.

$\Rightarrow$ So, TDA Not gives accurate stability.

FDA:

BP

NP



$e^{-s\tau} = e^{-j\omega}, \quad m = 1. \quad \angle\phi = (-\omega\tau).$

In FDA there is no any approximation.

* Compansators | Controllers:

$\Rightarrow$ It is required to get desired system. It is a simple electrical N/w which adds the poles and the zeros to the system in order to get the disired performance of the system.

* Steady State Analysis:

$\Rightarrow$ It is only valid for non linear, linear, time variant & time invariant system. It is define for dynamic system.

* System: Ch-1: TF, BD & SFG

⇒ (i/p) ⟶ [System] ⟶ Proper (o/p).
May (or) May not be desired o/p.

⇒ A System is a group of elements (or) a physical components arranged in a such a way that it gives the proper output to the given input.

⇒ A Proper output is may (or) may not be the desired output.

⇒ for. e.g.:

→ A FAN w/o Blades ⇒ Not a system
⇒ No proper o/p ⇒ No air flow.

→ A FAN w/o Regulator. ⇒ System
⇒ Proper o/p ⇒ Air flow.

→ A FAN with Regulator ⇒ Controlled system
⇒ Gives Desired o/p.

* Control System:

i/p ⟶ [Control system] ⟶ Desired o/p

⇒ A Control System is a group of physical Components arranged in a such a way that it gives the desired output by means of Control (or) regulate (or) Command either direct (or) indirect method to the given input.

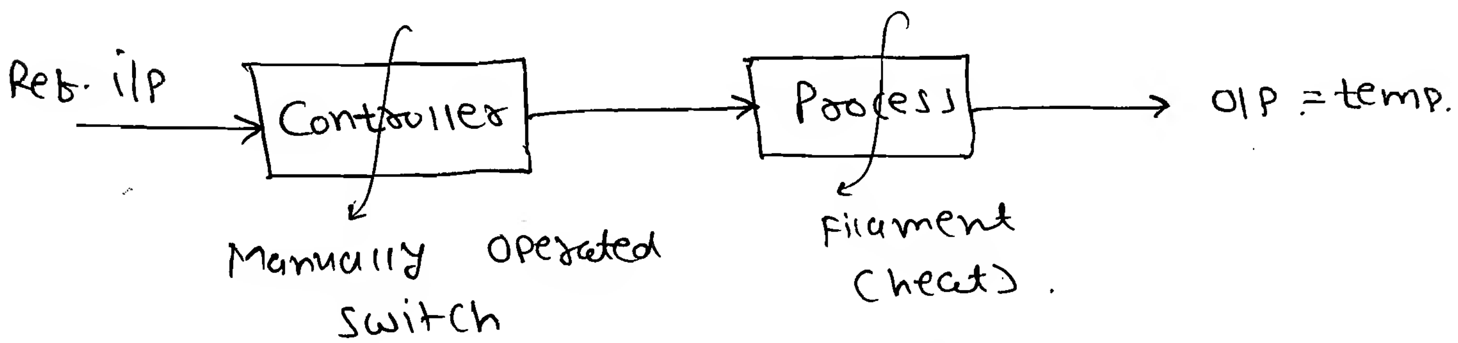
⇒ Control Systems are classified in two ways based on Controlling action.

i) open loop Control system (OLCS).

ii) Closed loop Control system (CLCS).

① Open Loop Control System : (manual):

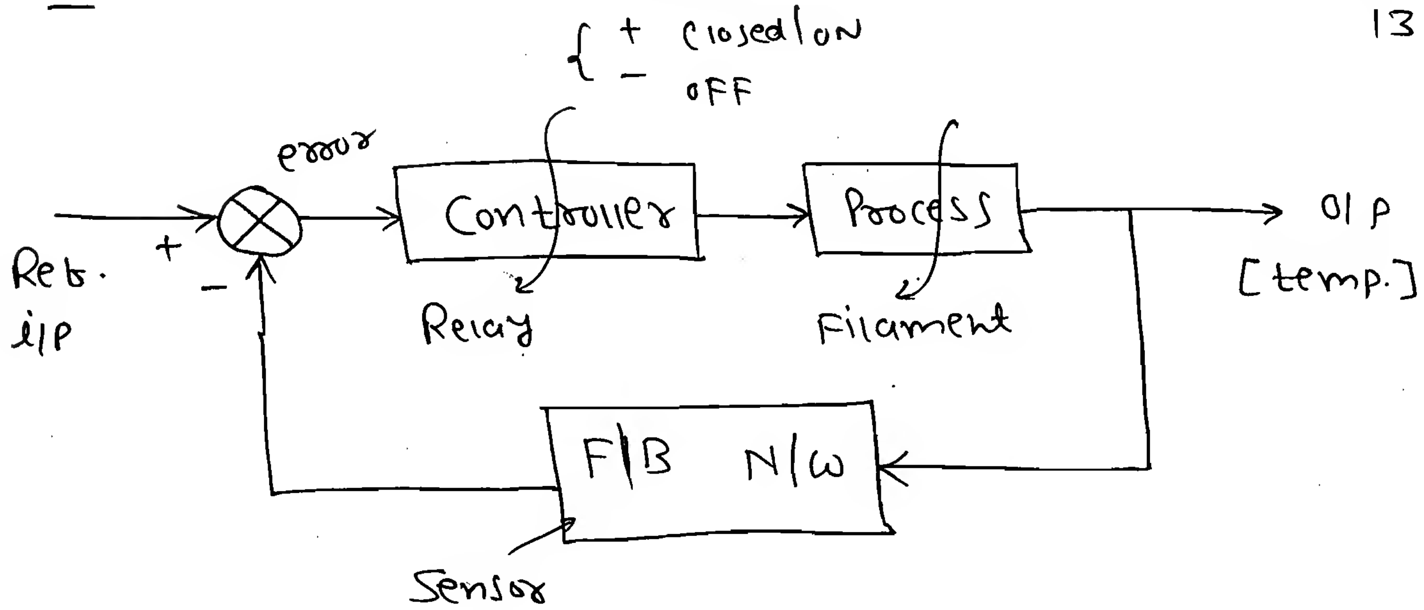
⇒



⇒ Ref. i/p is nothing but a desired o/p. [what we required or what we need].

⇒ A system in which the Controller action is completely independent of the output of the system is called open loop Control system. e.g. FAN, Lights, cooler, Traffic light.

(ii) Closed loop Control System: (Automatic):



⇒ A system in which the controller action depends on the o/p then it is called closed loop control system.

→ e.g.: AC, Refrigerator, Human beings, Automatic iron box and so on.

→ Any system which is having a sensor and provision to select the reference i/p.

* Feedback Network:

⇒ It is a property of the closed loop system which brings the o/p to i/p and compared with ref. i/p. So that appropriate control action formed to make the error equal to zero.

⇒ Error equal to zero, the system is stable which give the desired o/p.

⇒ The main Components in F/B N/W is R, L, C. the maximum gain of F/B N/W ratio is '1'.

⇒ The Best FB is unity (-ve) FB. Because the (-ve) FB improves the relative Stability. (loop gain > 0).

⇒ The Steady state errors are valid for only unity FB system. If non-unity FB System is given it should be Converted into unity FB.

⇒ The FB N/W may consist the transducer which Converts the energy from one form to another form.

* Transfer Function:

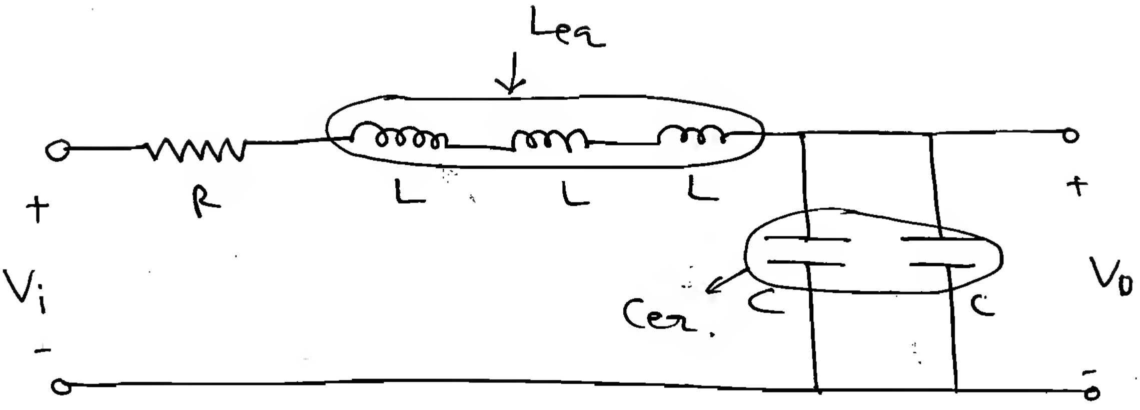
⇒ The transfer function is basically mathamatical equavalent model for the system.

⇒ The order of the transfer f^n represents the no. of storage elements (or) no. of the time constants.

Note: Whenever Same kind of elements Connected either series (or) parallel

it should be treated as a single Components.

for e.g.:



2nd order.

⇒ First defination of T.F.:

⇒ A T.F. of a Linear Time invariant system is defined as the ratio of Laplace transform of output to the Laplace transform of input with all initial conditions are zero.

$$T.F. = \left.\frac{L[O/P]}{L[I/P]}\right|_{I_i=0}$$

⇒ LTI system:

⇒ The LTI system is nothing but RLC ckt because the RLC Components gives the Linear Transfer characteresticS and the R,L,C Components values are not changes w.r.t. time.

⇒ In TF analysis the initial conditions must be zero because the output should not depends on the past history of the

System. It should depend on the Component Values and present i/p.

⇒ <u>Second</u> <u>Defn</u> <u>of</u> <u>T.F.</u>:

⇒ The TF of the LTI System is defined as Laplace transform of Impulse Response with all initial Conditions are zero.

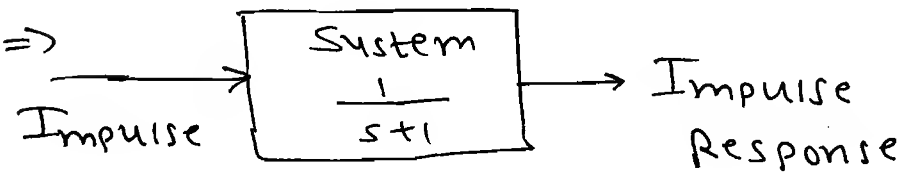
i.e. $$TF = \underbrace{L[\text{Impulse Response}]}\Big|_{I_i = 0.}$$

Sys Res | Natural Res | Free forced Res.

$$\Rightarrow T.F. = \frac{L[O/P]}{L[i/p]} = \frac{L[\text{Impulse Res}]}{L[\text{Impulse}]} = \frac{L[I.R.]}{1}$$

$(\because L[\delta(t)] = 1]$.

⇒



$$\frac{C(s)}{R(s)} = \frac{1}{s+1}$$

$R(s) = 1$

$$\therefore C(s) = \underbrace{\frac{1}{s+1}}_{\text{system comp.}} \cdot 1.$$

So, called sys. Res.

⇒ if we take $R(s) = \frac{1}{s}$. i.e. $r(t) = u(t)$.

$$\therefore c(s) = \frac{1}{(s+1)} \cdot \frac{1}{s}.$$

Response has input ~~cause~~ terms so it is not called sys. Response.

So, finding Sys. Res. we should take $R(s) = 1$.

⇒ The impulse Response gives the system behaviour (or) System characteristics because the Impulse Response Consist only system Parameters. No i/p term presents in the impulse Response. Hence the impulse Response is called system Response / Natural Response (or) Free forced Response.

⇒ If the signals are unit step, ramp (or) Parabolic then their response is called Forced Response.

* Transfer Function to Electrical N/w:

⇒ Any System basically defined in terms of OLTF.

⇒ The Standard form of the system is described as

Time constant form.

$$\boxed{G(s) = \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^n(1+s\tau_a)(1+s\tau_b)\cdots}}$$

⇒ K & τ are called system Parameters.

K: system Gain.

τ: system Time Constant.

n: Type-n System.

⇒ Type gives the no. of Poles at origin.

⇒ order gives the total no. of poles ~~and~~ ~~z~~ in s-plane.

Q Find the System gain, type and Order to the following system.

OL sys. $\frac{C(s)}{R(s)} = \frac{10(s+5)^2}{s^3(s+2)^2(s+10)}$ [Pole-zero form] *

Soln:

$$\frac{C(s)}{R(s)} = \frac{10 \times 25 \left(1+\frac{s}{5}\right)^2}{4 \times 10 \times s^3 \left(s/2+1\right)^2 \left(1+s/10\right)}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{6.25 \left(1+\frac{s}{5}\right)^2}{s^3 \left(1+\frac{s}{2}\right)^2 \left(1+\frac{s}{10}\right)}$$ [Time cons. form or standard form]

Type: → 3

Order: → 6

Gain: → 6.25

** ↓ H.B

$$\boxed{\text{Sys. gain } K = \frac{\text{Nr. Const}}{\text{Dr. const}}}$$

Q Find the type and order to the given CLTF of a unity feedback system.

$$\frac{C(s)}{R(s)} = \frac{2s+5}{s^5+4s^4+6s^3+7s^2+2s+5} = \frac{G(s)}{1+G(s)}$$

Soln: Here, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$\therefore \frac{G(s)}{1+G(s)} = \frac{2s+5}{s^5+4s^4+6s^3+7s^2+2s+5}$$

$$\therefore G(s)\left[s^5+4s^4+6s^3+7s^2+2s+5\right] = 2s+5 + G(s)\left[2s+5\right].$$

$$\therefore G(s) = \frac{2s+5}{s^5+4s^4+6s^3+7s^2}$$

$$\therefore G(s) = \frac{2s+5}{s^2\left(s^3+4s^2+6s+7\right)}.$$

So, order $\rightarrow$ 5.

Type $\rightarrow$ 2.

NOTE: H.B

$\Rightarrow$ The Type & order is not defined for closed loop TF. To get a type and order for CL sys. require OLTF of unity feedback system.

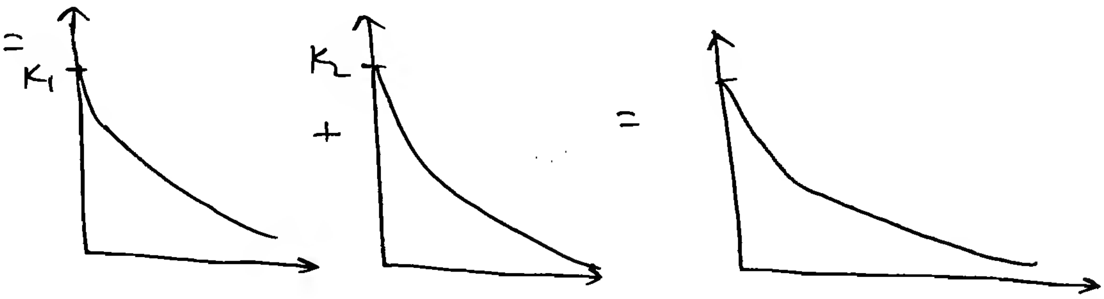
* Characterestics equation:

$$\Rightarrow \frac{C(s)}{\cancel{R(s)}\ 1} = \frac{(s-10)}{(s+1)(s+5)}.$$

$$= \frac{K_1}{(s+1)} + \frac{K_2}{(s+5)}.$$

$$\therefore \frac{C(s)}{R(s)} = K_1 e^{-t} + K_2 e^{-5t}.$$

$C(s) =$



⇒ Denominator terms decide the Chara. of the system not Numarator term. so for Chara. eqn we take denominator term equal to zero.

⇒ The Denominator of transfer function makes equal to zero then it is called Characterestic equation.

⇒ The chara. eqn gives the system behaviour (or) characterestics of the system.

→ For a CL system, the characterestics equation is $\boxed{1 + G(s).H(s) = 0.}$

⇒ The roots of Chara. eqn is called Poles.

* Pole:

⇒ The Pole is nothing but the negative of Inverse of system time constant at which magnitude of the TF becomes ∞.

$$S_p = -\frac{1}{\tau_a}, -\frac{1}{\tau_b}, \ldots \quad \Big| \; |TF| = \infty.$$

* Zero:

⇒ The zero is nothing but the negative of the inverse of the system time const. at which magnitude of the TF become 0.

$$S_z = -\frac{1}{\tau_1}, -\frac{1}{\tau_2}, \ldots \qquad |TF| = 0.$$

H.B

⇒ The Pole can affect the system response and system stability but not the zero.

* Time Constant:

⇒ The time constant gives the system behaviour. If the time constant is very very large then it is called slow response system. Because it takes the large time to reach the steady state.

→ Practically any ~~ste~~ system takes the 5τ to reach the steady state.

$$\boxed{\tau = -\frac{1}{\text{Real part of the Dominant pole}}}$$

H.B.

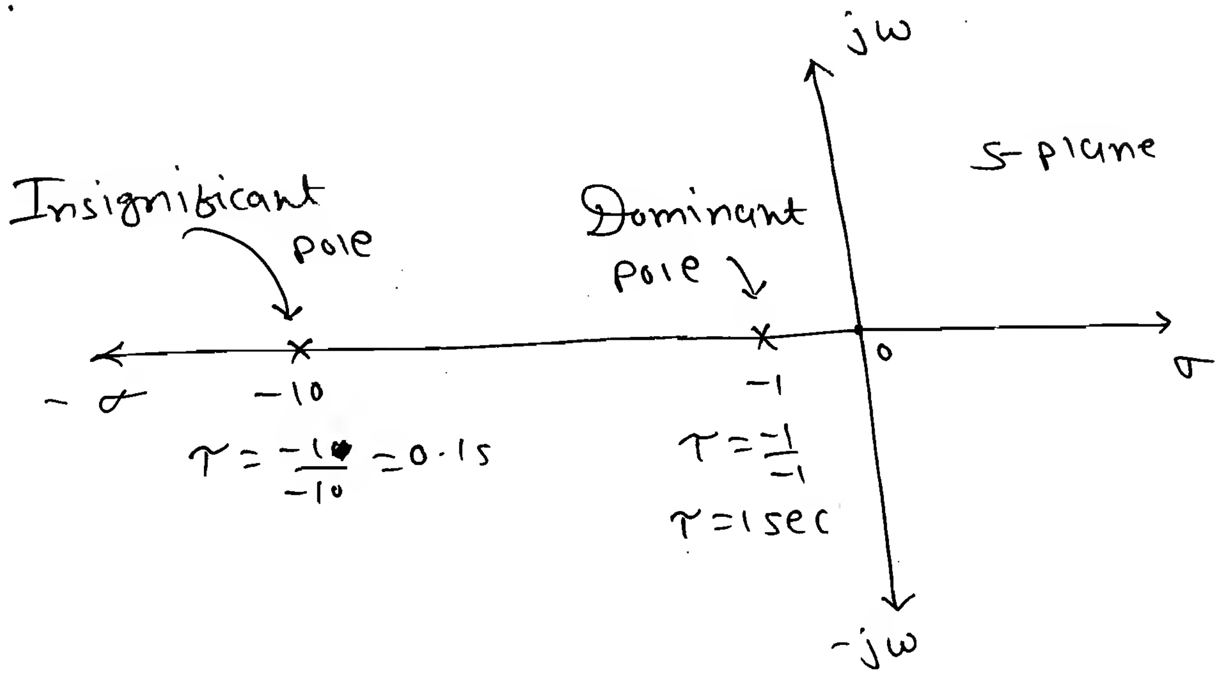
⇒ **Dominant Pole:** ← H.B

⇒ The pole which is very close to the imaginary axis is called as dominant pole.

Q ① Find the equivalent 1^{st} order system.
② Find the system time constant for

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}.$$

Soln:



⇒ Insignificant pole has less time constant so good performance and hence best pole.

⇒ Dominant pole has large time constant so bad pole. It affect the system. So we have to compensate it by adding zero at same position. So ~~that~~ we discuss only for DP.

If
⇒ **H.B** $\boxed{\text{Insignificant Pole} \le 5 \cdot \text{Times of Dominant pole.}}$

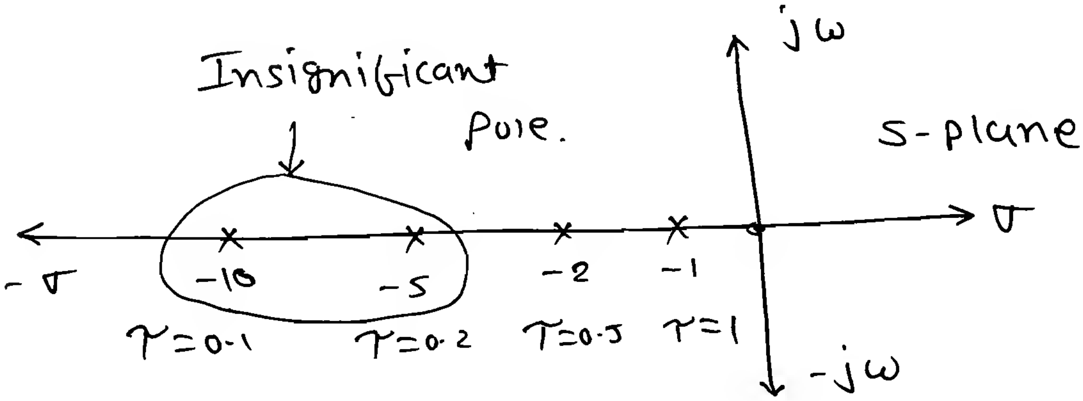
then only it is called insignificant pole.

23

i.e here $-10 \le 5(-1)$

$\Rightarrow -10 < -5$ ✓

Eg.



$$Ins(\tau) \le \frac{DP(\tau)}{5}$$

$$Ins(\tau) \le \left(\frac{1}{5} = 0.2\right)$$

* Insignificant Pole:-

→ The Poles which lies in the left most side.

→ The insignificant pole time Constant must be less than (OR) equal to 5 times of the dominant pole time constant that means insignificant pole.

$$\boxed{Isp(\tau) \le \frac{DP(\tau)}{5}}$$ ⇐ H.B

→ The best pole is the insignificant pole.

because it gives the very quick response and more relatively stable. Because of the dominant pole the system response become the slow and the system becomes less relatively Stable.

⇒ The insignificant Poles are neglected because even if insignificant poles are neglected there is no much Change in the system Response.

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}.$$

System Response | $R(s)=1$.

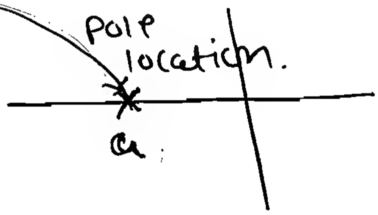
$$\therefore C(s) = \frac{1}{(s+1)(s+10)}.$$

$$\therefore C(s) = \frac{1}{9(s+1)} - \frac{1}{9(s+10)}.$$

$$\xrightarrow{ILT} c(t) = \underbrace{\frac{1}{9}e^{-t}}_{\substack{\downarrow \\ T=1s \quad DP\ Res.}} \; \underbrace{-\frac{1}{9}\cdot e^{-10t}}_{\substack{\downarrow \\ ISP\ Res. \quad T=0.1sec.}}$$

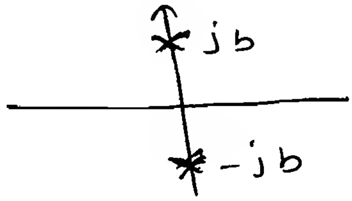
$$\Rightarrow L[e^{-at}] = \frac{1}{s+a}$$

Pole location.



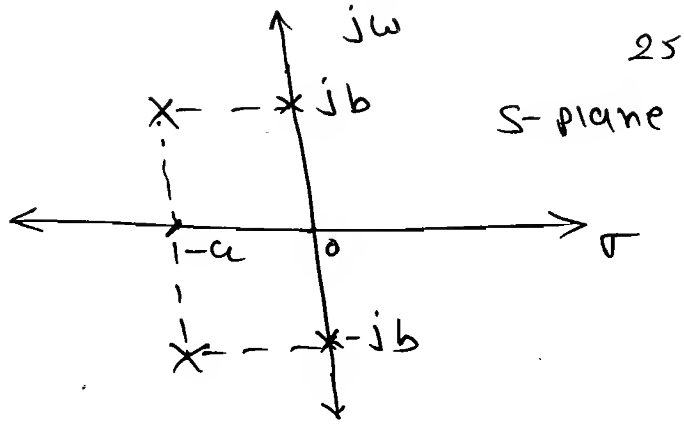
Real part of pole

$$\Rightarrow L[\sin(\text{or})\cos bt] = \frac{b \text{ or } s}{s^2+b^2}.$$

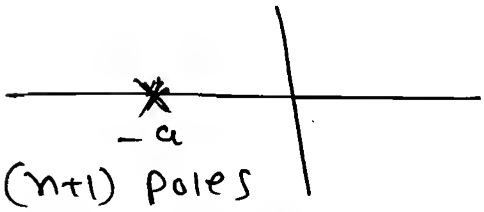


Img part of pole.

$\Rightarrow L[e^{-at} \cdot \sin bt] \Rightarrow$



$\Rightarrow L[t^{n} \cdot e^{-at}] \Rightarrow$



(n+1) poles are repeated.

→ In the response exponential powers are Real part of the poles, sine or cos function are Imaginary part of poles. The t term represents the Repeated nature of the poles.

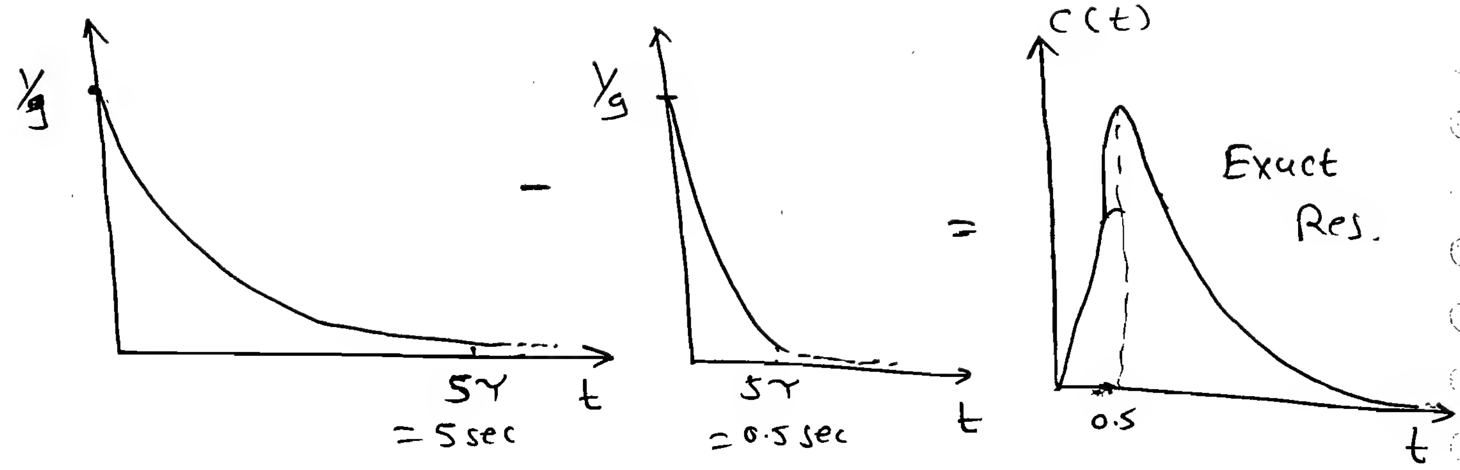
⇒ To get the system time constant from the response, compare the response with $e^{-t/\tau}$.

⇒ The system time constant is nothing but the dominant pole time constant and it should have the largest value.

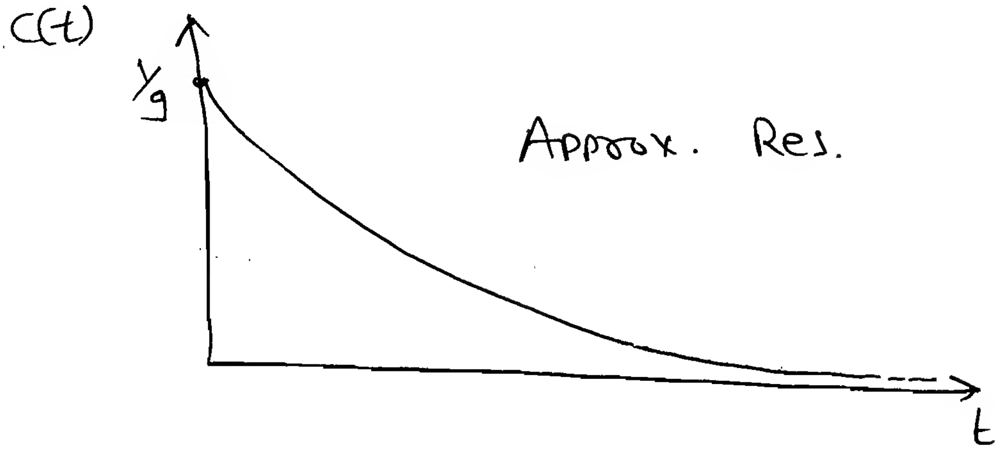
⇒ $\dfrac{C(s)}{R(s)} = \dfrac{1}{(s+1)(s+10)}$ (Pole-zero form).

never neglect pole directly in pole-zero form.

H.B.



|||



$$\Rightarrow \quad \frac{C(s)}{R(s)} = \frac{1}{10(s+1)(1+0.1s)}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{0.1}{(s+1)}, \quad \tau = 1\text{ sec.}$$

System Res. $\Rightarrow R(s) = 1 \Rightarrow \quad c(t) = 0.1e^{-t}$.

NOTE: ✓ (h.B).

⇒ The Insignificant Pole must be neglected only in the time constant form.

Q Find the equivalent TF of the following system $\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)(s+100)}$.

(a) $\frac{1}{s+1}$ (b) $\frac{1}{(s+1)(s+10)}$ (c) $\frac{0.01}{(s+1)(s+10)}$ (d) ✓ $\frac{0.001}{(s+1)}$.

* While finding,

Sys. Response.

Sys. Stability.

Sys. Time Constant

We consider only Poles not zeros.

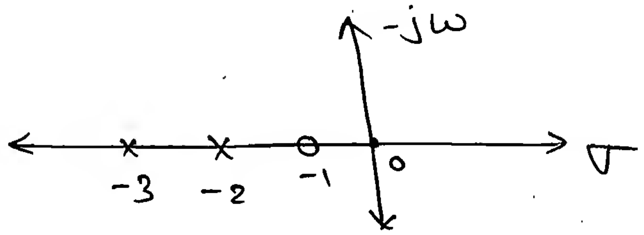
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{(S+1)}{(S+2)(S+3)}.$$

Sys. Resc ; $R(s) = 1$.

$$\therefore C(s) = \frac{(S+1)}{(S+2)(S+3)}.$$

$$\therefore C(s) = \frac{-1}{(S+2)} + \frac{2}{(S+3)}.$$

$$\xrightarrow{ILT} c(t) = \left(-e^{-2t} + 2e^{-3t}\right).$$



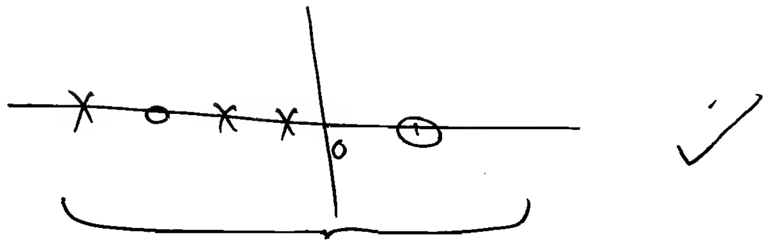
$\Rightarrow$ While finding System Response, system Stability, system time Constant we Consider only Poles but not zeros because the system Response Consist only the Poles response terms there is no zeros response term exist in the System Response.

⇒ Stability ⇒ $t = \infty$ ⇒ sys. Res.

'∞' value ⇒ Unstable

Finite value ⇒ Stable.

⇒



Stable (∵ no Pole at RHS).

⇒ $G(s) = K \dfrac{(\text{OL zeros})}{(\text{OL Poles})}$

CL sys: → $\text{CLTF} = \dfrac{\text{CL Zero}}{\text{CL Poles}}$

CL sys. → $G(s) = \dfrac{K(\text{OL zeros})}{(\text{OL Poles})}$; $H(s) = 1$

CLTF → $\text{CLTF} = \dfrac{K(\text{OL zeros})}{(\text{OL poles}) + K(\text{OL zeros})}$

⇒ The OL zeros never affect the OL Stability.

⇒ The CL Zeros never affect the CL stability.

⇒ The OL Zeros affect the CL Stability because the CL Poles are nothing but

the Sum of the OL Poles & OL Zeros with the function of Sys. gain K.

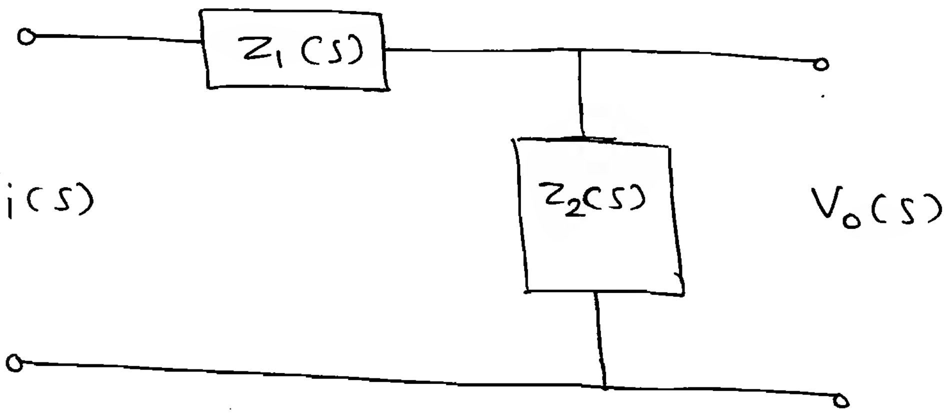
⇒ NOTE:

(i) To get the OLTF from the CLTF, Subtract numerator in the denominator when the feedback is unity.

(ii) To get the CLTF from OLTF, Add the numerator ~~from~~ in the dinominator when the F/B is unity.

* Transfer function of the Electrical N/ws:

⇒

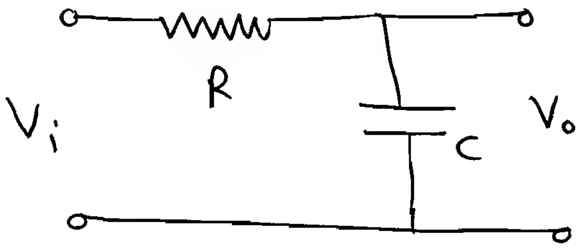


$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\text{Impedence across O/P}}{\text{Total CKt impedence}}$$

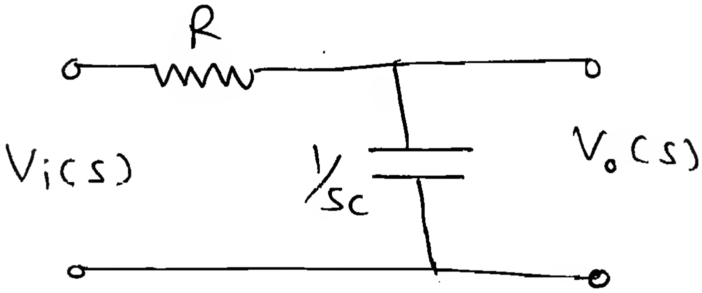
$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}}$$ ← H.B

[Q] (a) Find the TF to the given electrical N/ws & locate the poles in S-plane.

(b) Find the System Response.



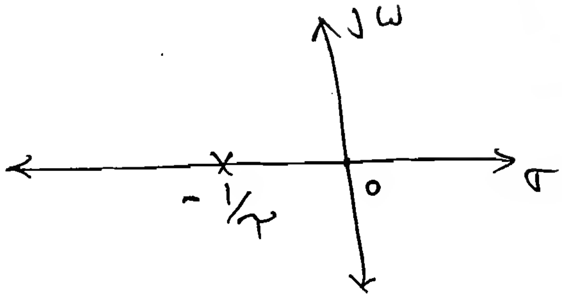
Soln:



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sCR}.$$

$\tau = RC$

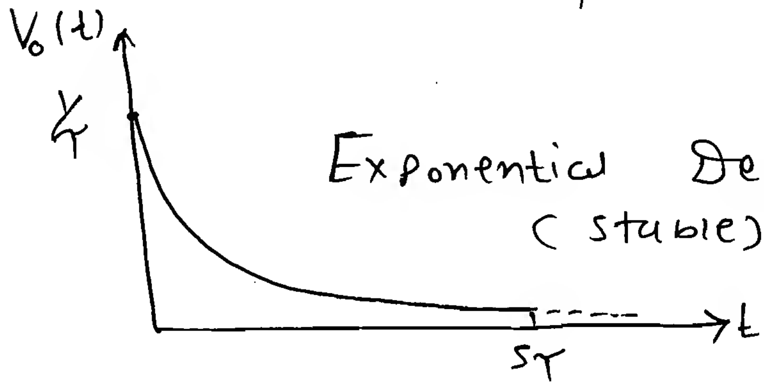
$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + s\tau}$$



$\Rightarrow$ For Sys. Res. $V_i(s) = 1$.

$$\therefore V_o(s) = \frac{1}{\tau(s + 1/\tau)}.$$

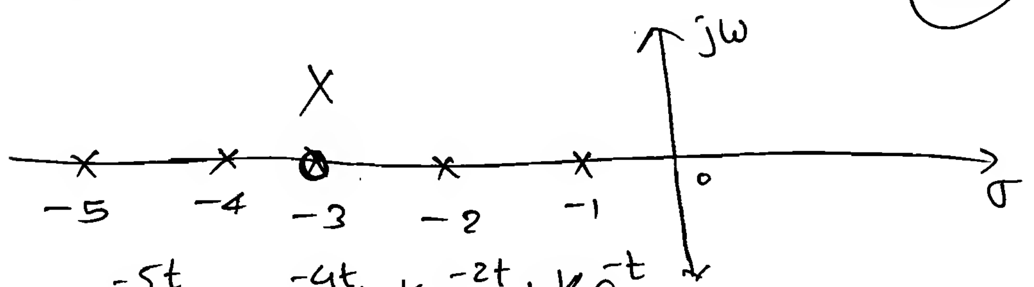
$$\xrightarrow{ILT} V_o(t) = \frac{1}{\tau} e^{-t/\tau}.$$



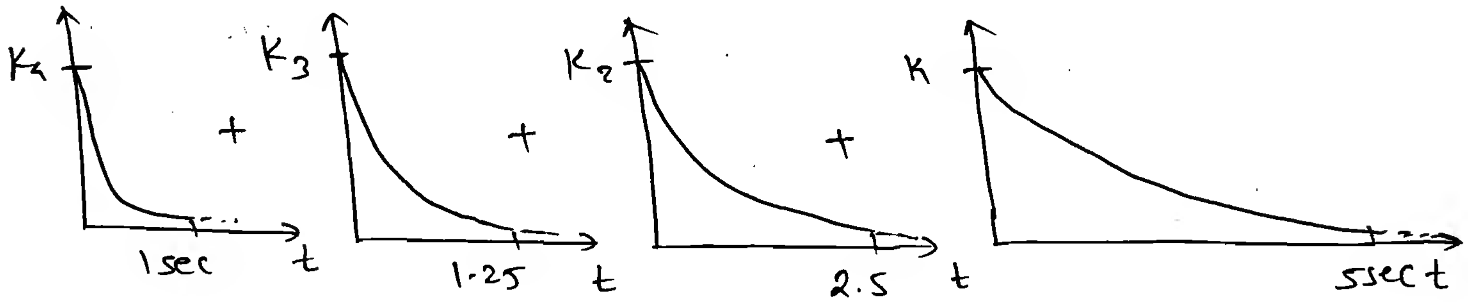
Exponential Decay (stable)

NOTE:

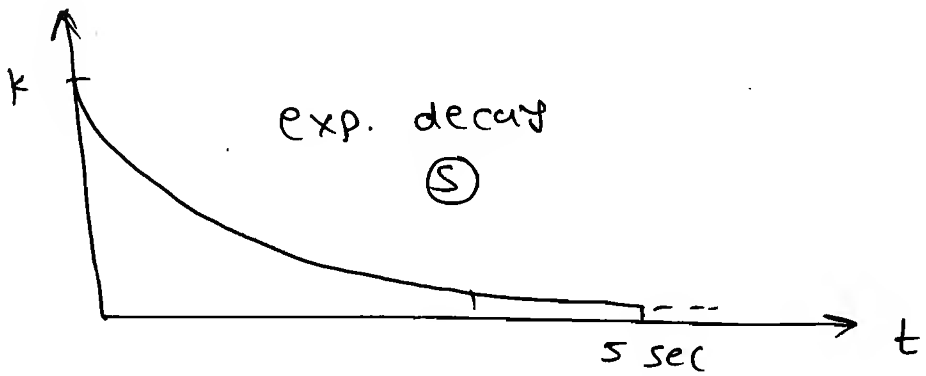
⇒ If one (or) more poles lies in the left of s-plane at different locations then the system response is exponential decay irrespective of position of zeros, the system is stable. ⇑ (H.B.)



$K_4 e^{-5t} + K_3 e^{-4t} + K_2 e^{-2t} + K e^{-t}$



=



* Stability :

⇒ The movement of the pole in the s-plane is nothing but varying the system components (R, L, C).

⇒ Absolutely Stable System means the System is stable for all the values of the system Parameters (or) system Components like 'K' from '0 to ∞'.

⇒ Conditional Stable System means the System is stable for certain range of System Components like 'K' from 0 to 100.

⇒ Addition of Poles & zeros to TF means adding RLC's Components to the system.

⇒ The RLC Components added to the System in a two ways.

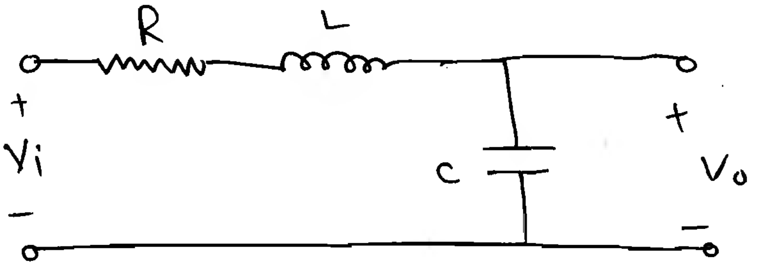
① series Connection.

② Parallel Connection

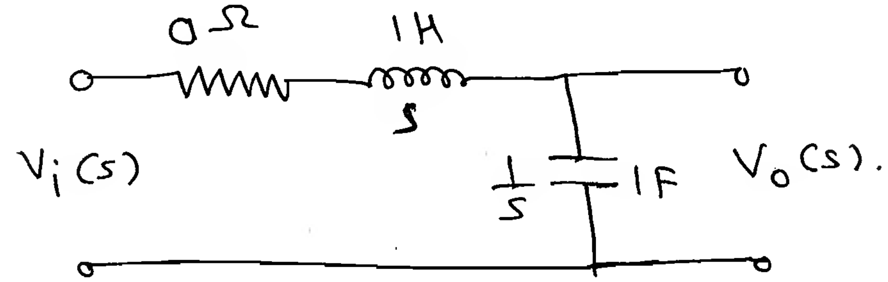
⇒ In series Connection the RLC components are added in a forward path.

⇒ In Parallel Connection the RLC Components added in a feedback path.

$\boxed{Q-1}$ Find the T.F. to the given electrical N/W and Locate the poles in the s-plane by considering $R=0$, $L=1H$, $C=1F$.



Soln:



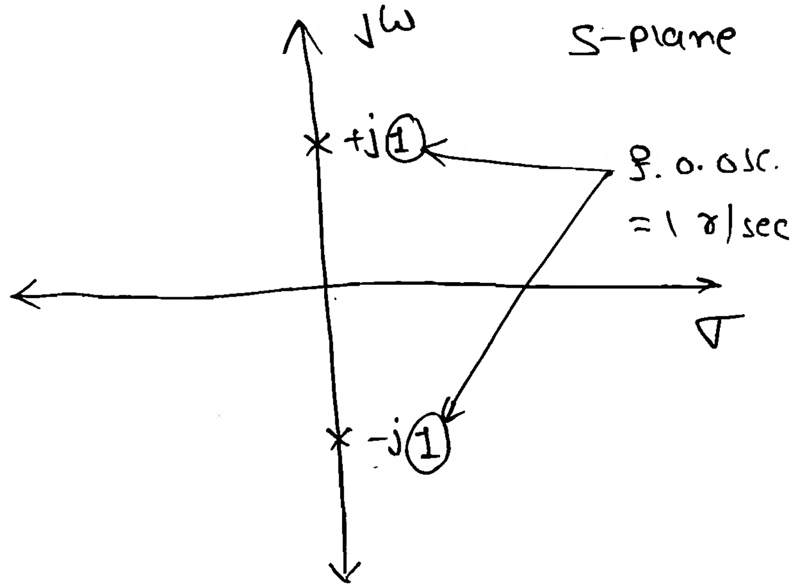
$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{1}{s^2LC + sCR + 1}.$$

$\Rightarrow$ $R = 0\,\Omega$, $L = 1H$, $C = 1F$.

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + 0 + 1} = \frac{1}{s^2+1}.$$

Poles: $s^2 + 1 = 0 \Rightarrow s = \pm j$.

$\Rightarrow$ Non-Repeated Poles on $j\omega$ axis, system marginally stable.

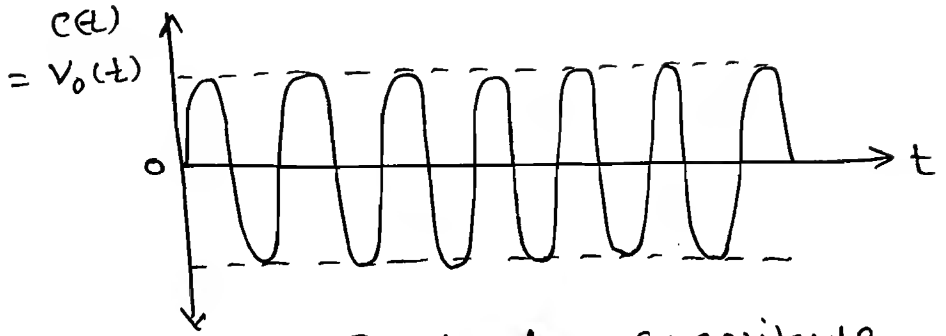


$$\Rightarrow V_o(s) = \frac{1}{(s^2+1)} \cdot V_i(s).$$

for system response $V_i(s) = 1$.

$$\therefore \quad V_o(s) = \frac{1}{s^2+1}$$

$$\Rightarrow V_o(t) = \sin t.$$



Constant amplitude & freq. of oscillation

Undamped oscillation i.e. Marginally Stable.

⇒ When the Poles lies on imaginary axis which are non-repeated then the system response is constant amplitude and freq. of oscillation which are called undamped oscillation.

⇒ Any system which produce undamped oscillation is called undamped ~~undamped~~ System. and the system becomes marginal stable.

$\boxed{\text{Q}}$ Repeate the above Problem by Considering $R = 1\,\Omega$, $C = 1F$, $L = 1H$.

Soln:

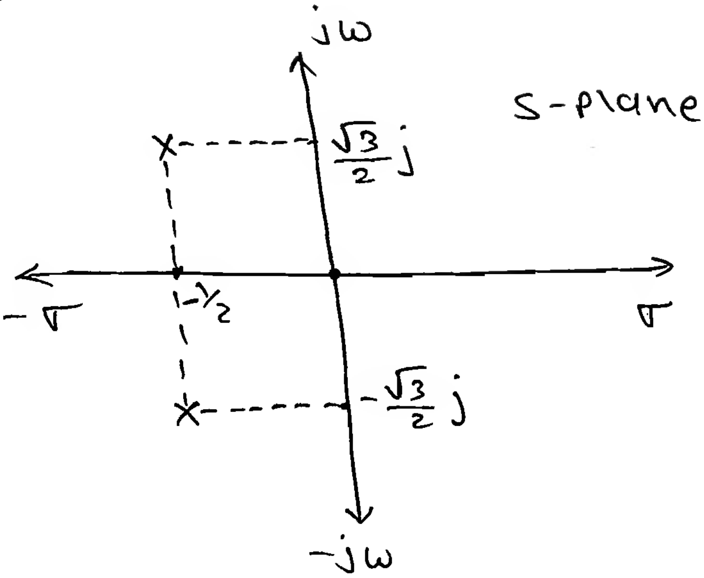
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + sCR + 1}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+s+1}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+s+\frac{1}{4}+3/4}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

Poles: $s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$.



$\therefore$ Time Constant

$$\boxed{\tau = \frac{-1}{-1/2} = 2 \text{ sec.}}$$

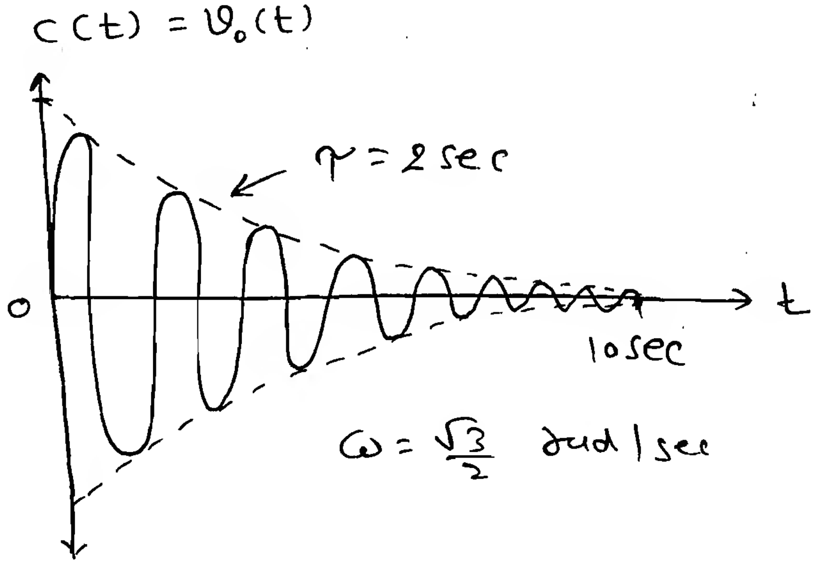
$$\therefore j\omega = \frac{\sqrt{3}}{2}j$$

$$\therefore \boxed{\omega = \frac{\sqrt{3}}{2} \text{ rad/sec.}}$$

NOTE: H.B.

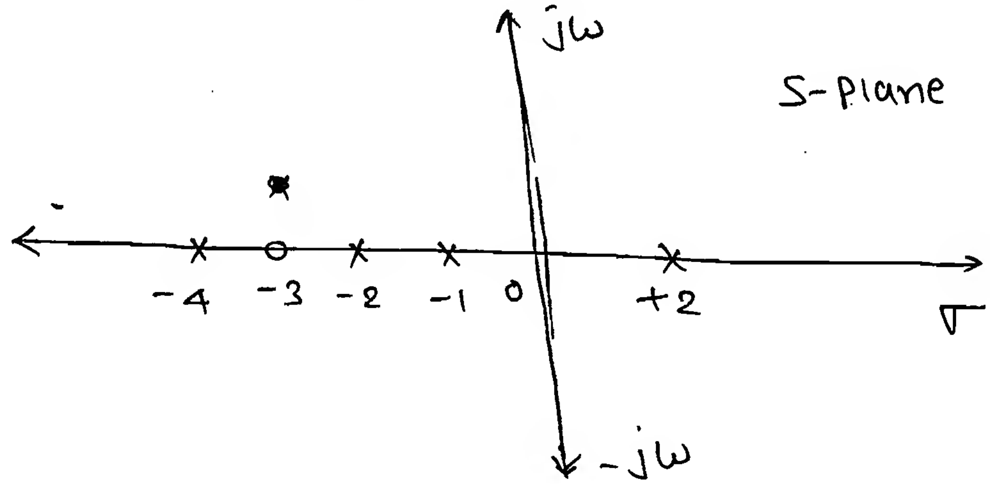
$\Rightarrow$ In Complex Conjugate poles, the real part gives the system time constanat and imaginary part gives the freq. of oscillation.

$$v_o(t) = c(t) = \frac{2}{\sqrt{3}} \cdot e^{-t/2} \cdot \sin\frac{\sqrt{3}}{2}t$$

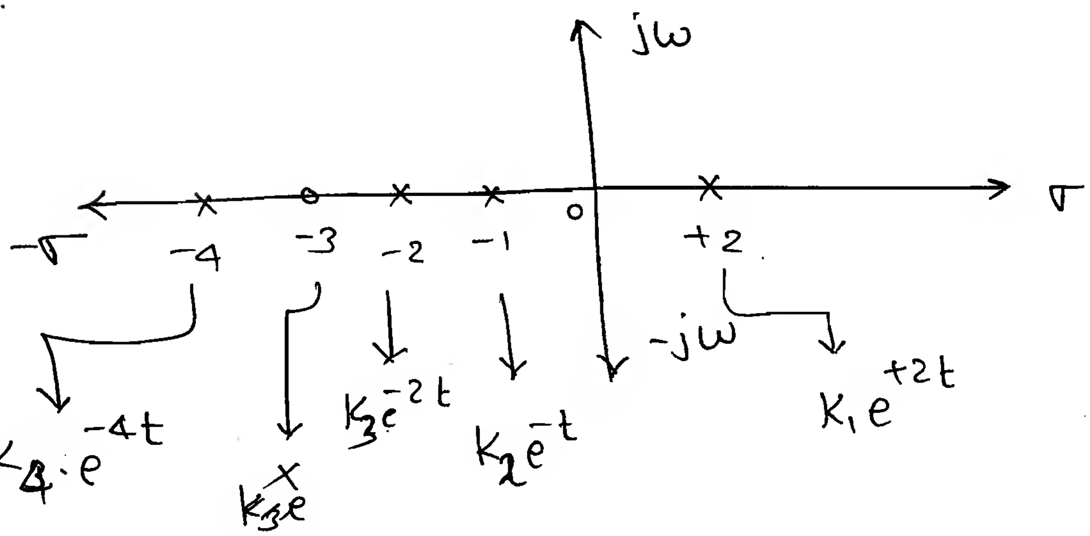


⇒ whenever the poles are complex conjugate in the left of S-plane then the system response is exponetically decay and freq. of oscillation which are called damped oscillations. Any system which produce damped oscillations is called under damped system and the system is stable.

[Q] Find the system time const. and system response to the given poles location in the s-plane.

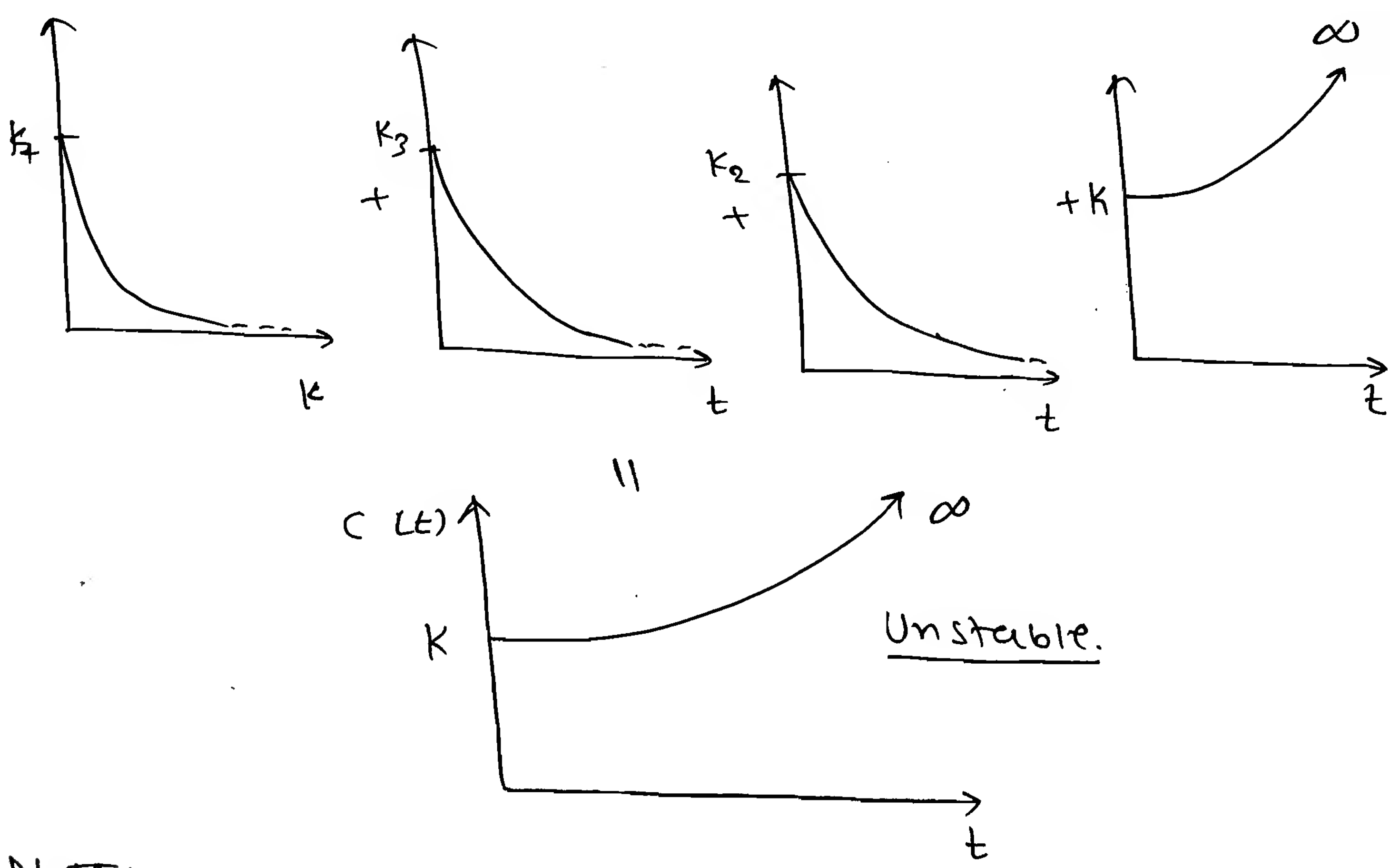


Sol^n:



$$\Rightarrow \frac{C(s)}{R(s)} = \frac{(s+3)}{(s+1)(s+2)(s+4)(s-2)}$$

→ A given system is un-stable and the time const. is not defined. The time constant defined for only stable system.



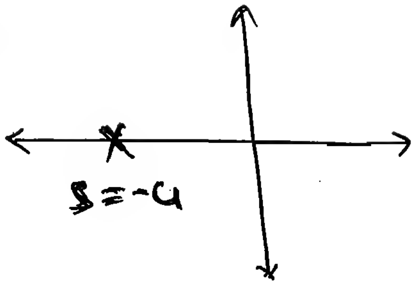
Unstable.

NOTE:

⇒ Whenever one (or) more poles lies in the right of the s-plane at different locations on the real - axis then the system is un-stable because the system response expential rised to infinity.

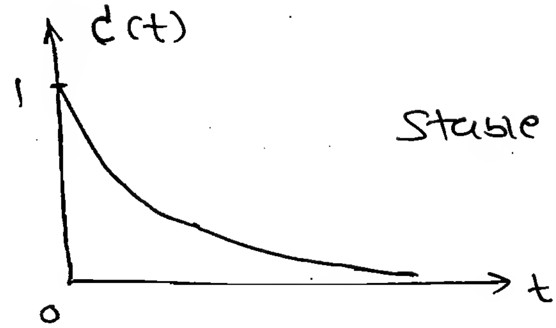
⇒ The system response follows the i/p then the system become stable.

*



Draw the Res.

$\Rightarrow$



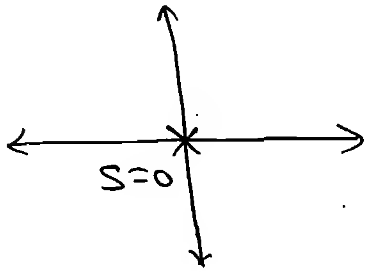
Stable

TF : $\frac{1}{s+a}$

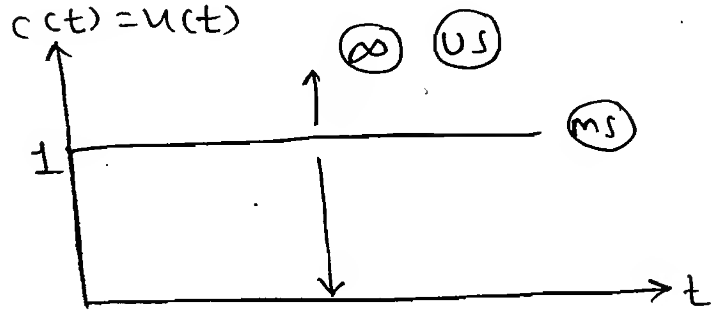
Sys. Res : $1 \cdot e^{-at}$

Sys. Response follows the i/p.

*



$\Rightarrow$



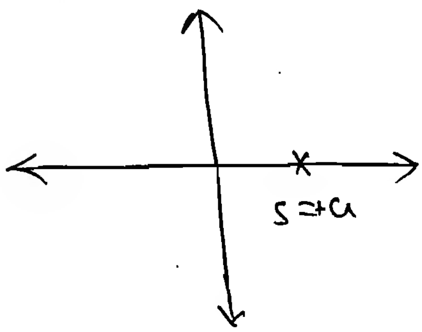
$\rightarrow$ TF = $\frac{1}{s}$.

$\Rightarrow c(t) = u(t)$.

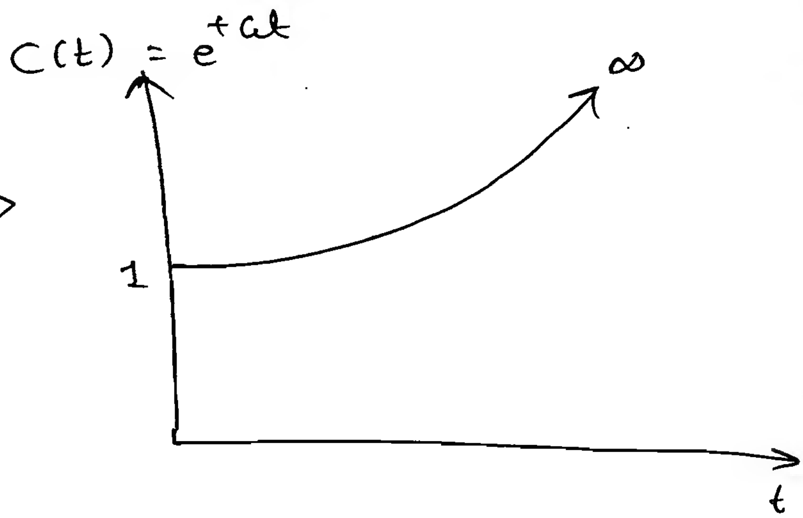
Const. Amp & f.o.o.

MS

*



$\Rightarrow$



TF = $\frac{1}{s-a}$.

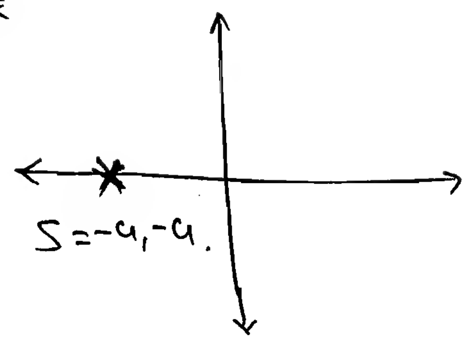
$c(t) = 1 \cdot e^{+at}$

Unstable

exponentially raised

So, unstable system.

*

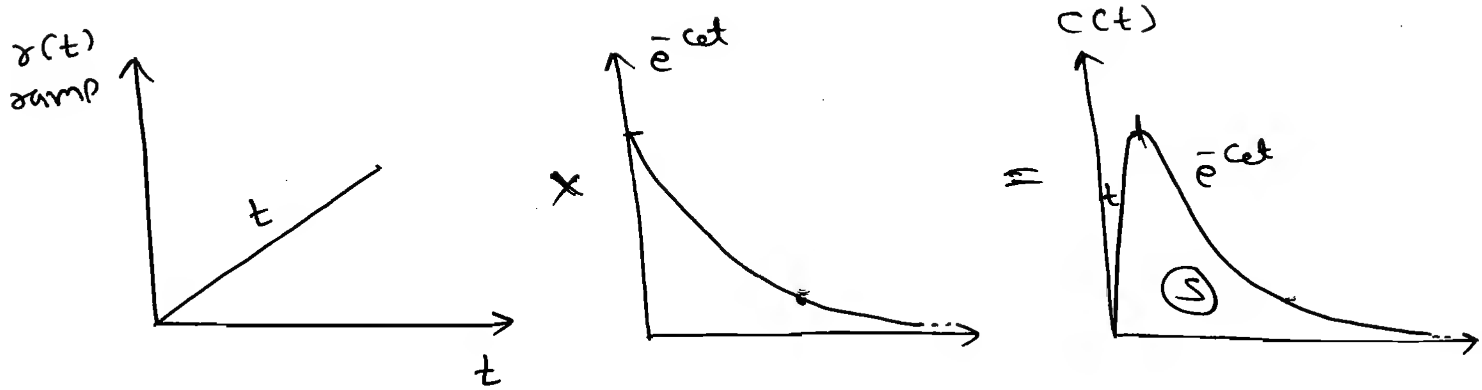


$$T.F. = \frac{1}{(s+a)^2}$$

$$c(t) = t \cdot e^{-at}.$$

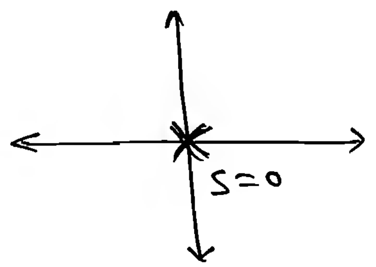
$t=0 \Rightarrow 0$ (t- term).

$t=\infty \Rightarrow 0$ (exp - term).

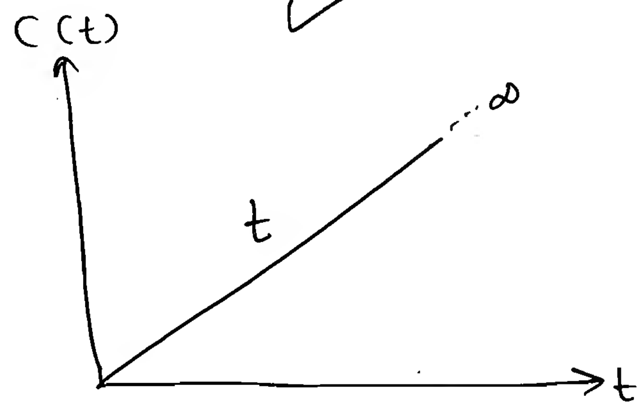


$\Rightarrow$ for Lower value of t, t terms dominant and for higher value of t, $\bar{e}^{at}$ term dominant.

*



$\Rightarrow$



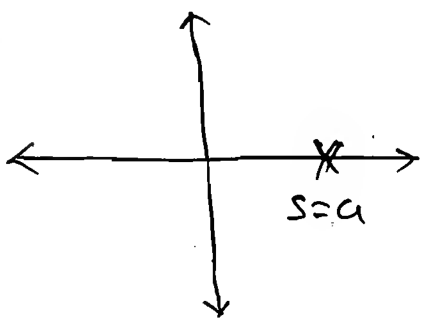
$$T.F. = \frac{1}{s^2}$$

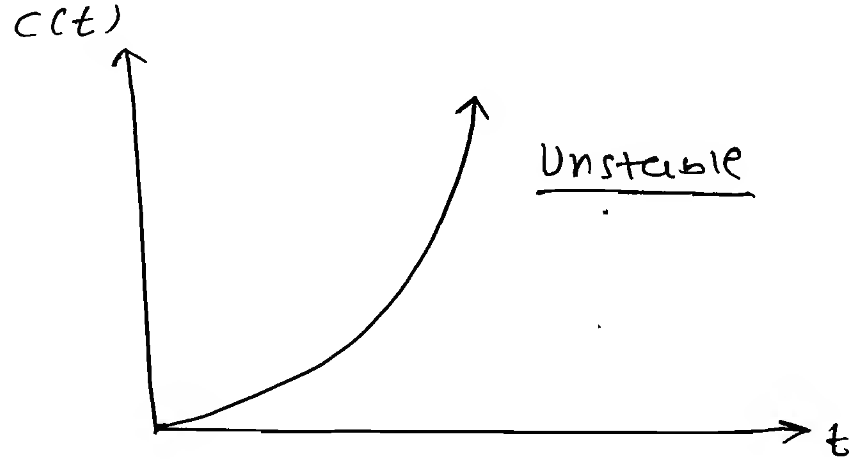
$$c(t) = t \cdot u(t).$$

Unstable.

Repeated pole on jw axis unstable.

*





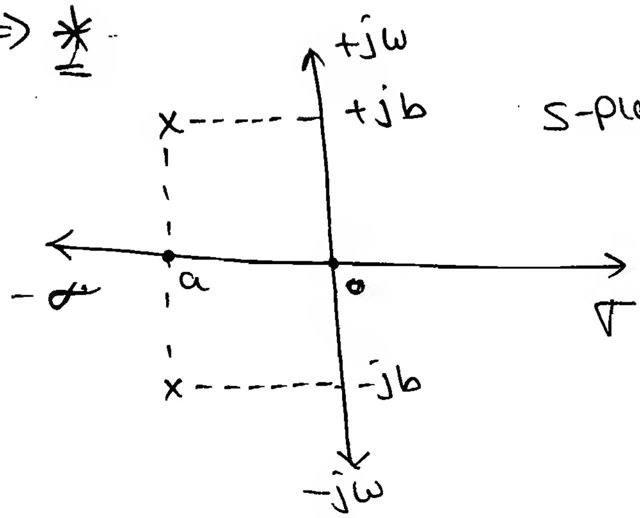
<u>Unstable</u>

$$T.F. = \frac{1}{(s-a)^2}$$

$$c(t) = t \cdot e^{at}$$

* Complex Conjugate Poles:

⇒ *

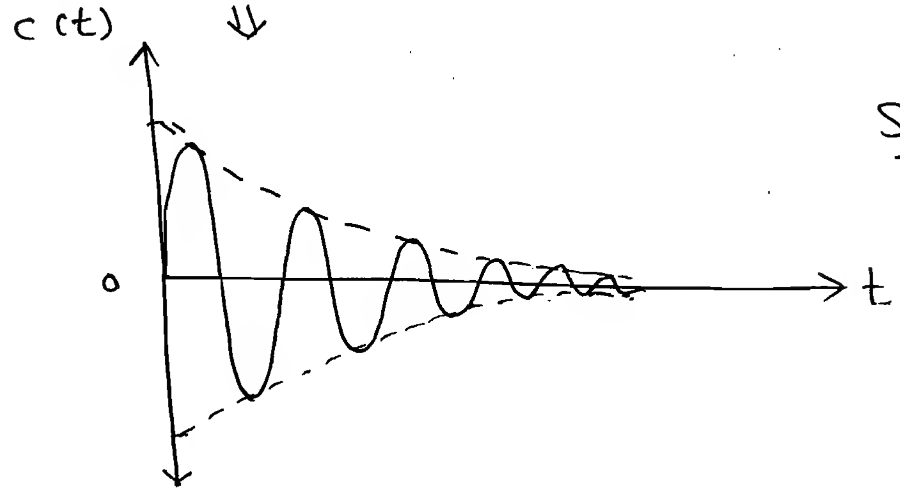


s-plane

$$TF = \frac{1}{(s+a)^2 + b^2}$$

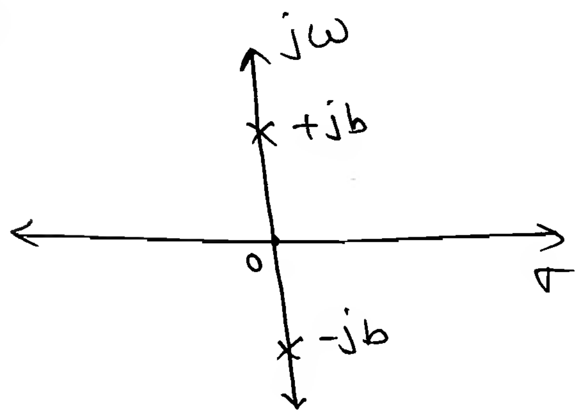
$$c(t) = \frac{1}{b} \cdot e^{-at} \cdot \sin bt.$$

⇓



Stable.

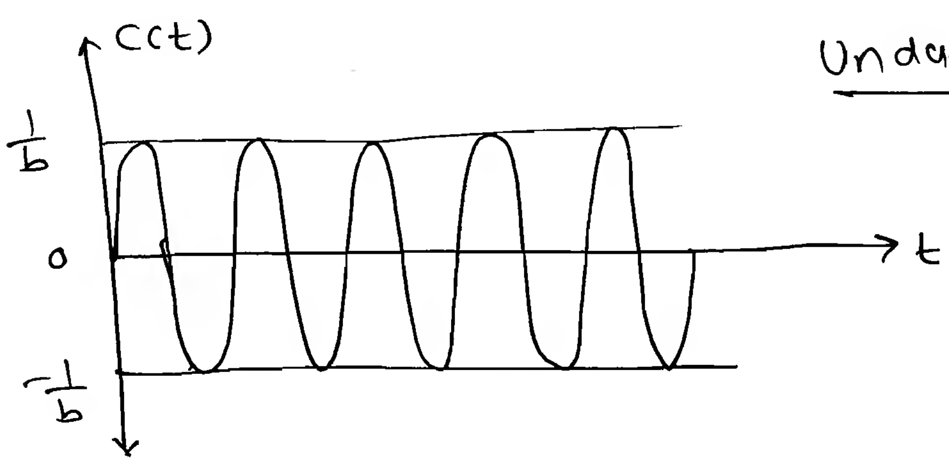
*



$$TF = \frac{1}{s^2+b^2}.$$

$$c(t) = \frac{1}{b} \cdot \sin bt.$$

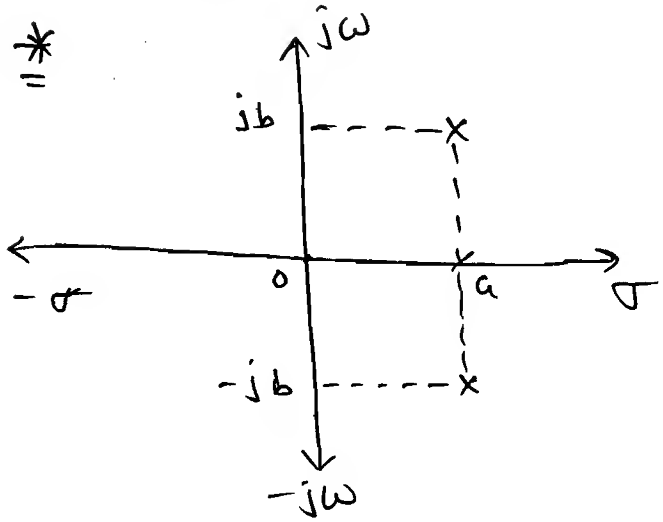
⇒



Undamped

→ MS

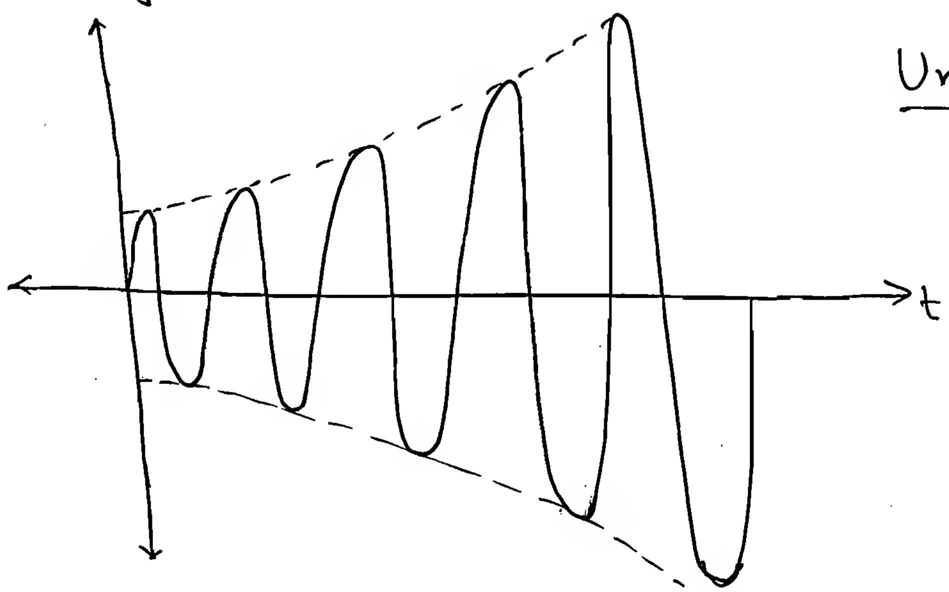
*



$$T.F. = \frac{1}{(s-a)^2 + b^2}$$

$$c(t) = \frac{1}{b} \cdot e^{at} \cdot \sin bt.$$

⇒



Unstable

⇒ S-plane of TF — Sys. Res.

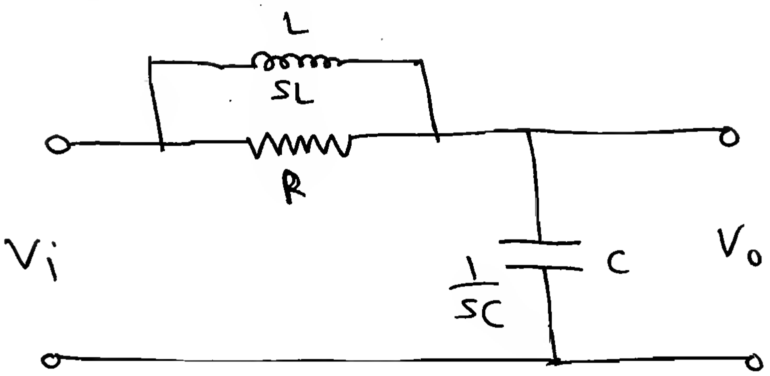
① Real Part ⟶ Exp. term.

② Img. part ⟶ Sine (or) cos term
(cos will come whenever there is a zero at origin)

③ (Real + Img) ⟶ Product of exp. and sine (or) cosine term.

④ (Repeated Pole) ⟶ Product of t and exp. term.

Q. Find the TF to the electrical N/W.



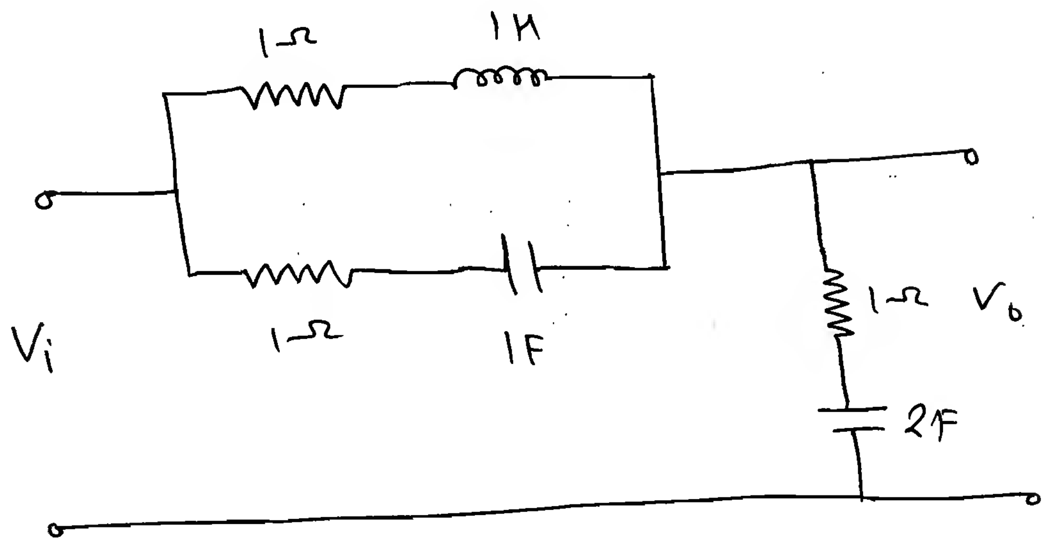
Soln:

$$SL \parallel R = \frac{RSL}{R+SL}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC}}{\frac{1}{SC} + \frac{RSL}{R+SL}}.$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{R+SL}{R+SL+S^2RLC}}$$

Q-2



Soln:

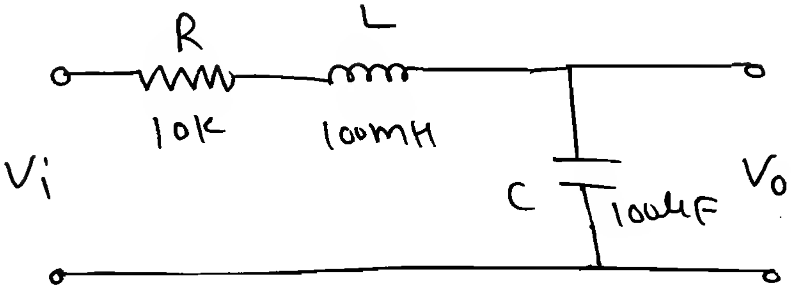
$$\left(1+\frac{1}{s}\right) \parallel (1+s)$$

$$= \frac{\left(1+\frac{1}{s}\right) \times (1+s)}{1+\frac{1}{s}+1+s}$$

$$= \frac{(1+s+\cancel{\frac{1}{s}+1})}{(1+\cancel{\frac{1}{s}+1+s})} = 1.$$

$$\therefore \frac{V_0(s)}{V_i(s)} = \frac{1 + \frac{1}{2s}}{1 + 1 + \frac{1}{2s}} = \frac{2s+1}{4s+1}.$$

Q-3



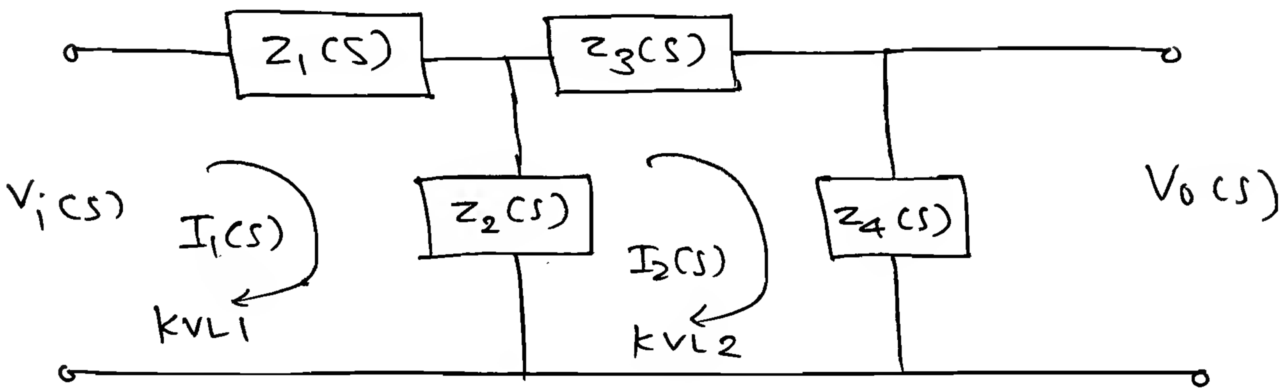
Soln:

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{s^2LC + sCR + 1}$$

$$= \frac{1}{s^2 \times 10^{-5} + s + 1}$$

$$\therefore \boxed{\frac{V_0(s)}{V_i(s)} = \frac{10^5}{s^2 + 10^5 s + 1}}$$

Q



Soln: By KVL,

$$V_i(s) = Z_1(s) \cdot I_1(s) + Z_2(s) \cdot [I_1(s) - I_2(s)]$$

$$V_i(s) = [Z_1(s) + Z_2(s)]\, I_1(s) - Z_2(s) \cdot I_2(s). \quad -①$$

By KVL,

$$Z_3(s) \cdot I_2(s) + Z_4(s) \cdot I_2(s) + [I_2(s) - I_1(s)]\, Z_2(s) = 0.$$

$$\therefore [Z_3(s) + Z_4(s) + Z_2(s)]\, I_2(s) - Z_2(s)\, I_1(s) = 0 \quad -②$$

$$\Rightarrow \quad V_o(s) = I_2(s) \cdot Z_4(s).$$

$$\begin{bmatrix} V_i \\ 0 \end{bmatrix} = \begin{bmatrix} Z_1(s) + Z_2(s) & -Z_2(s) \\ -Z_2(s) & Z_2(s) + Z_3(s) + Z_4(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}.$$

→ By using Crammer's rule,

$$\rightarrow \quad I_2 = \frac{\Delta_2}{\Delta}$$

$$\cancel{I_{z_4}} \ne$$

$$I_2 = \frac{\begin{vmatrix} Z_1 + Z_2 & V_i \\ -Z_2 & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 + Z_4 \end{vmatrix}}.$$

$$\therefore \quad I_2 = \frac{+V_i \cdot Z_2}{(Z_1+Z_2)(Z_2+Z_3+Z_4) - Z_2^2}.$$

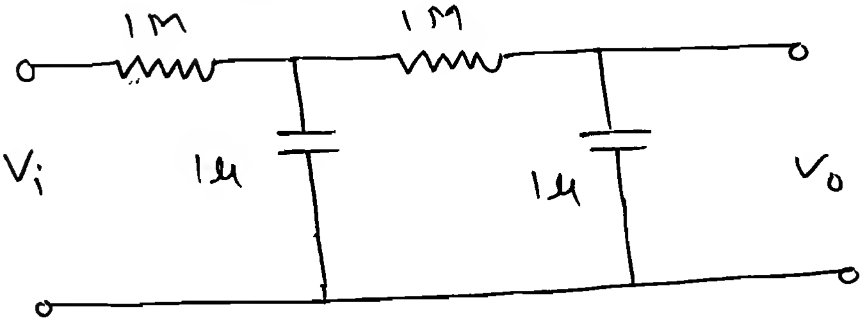
$$\therefore \quad I_2 = \frac{V_i \cdot Z_2}{Z_1 \cdot Z_2 + Z_1 \cdot Z_3 + Z_1 \cdot Z_4 + \cancel{Z_2^2} + Z_2 \cdot Z_3 + Z_2 \cdot Z_4 - \cancel{Z_2^2}}$$

$$\therefore \quad I_2 = \frac{V_i \cdot Z_2}{Z_1 \cdot (Z_2 + Z_3 + Z_4) + Z_2 \cdot (Z_3 + Z_4)}.$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2 \cdot Z_4}{\cancel{Z_1} \cdot \xi \; Z_1(Z_2+Z_3+Z_4) + Z_2(Z_3+Z_4)}}$$

↑ H.B.

[Q] Find the TF.



Sol^n:

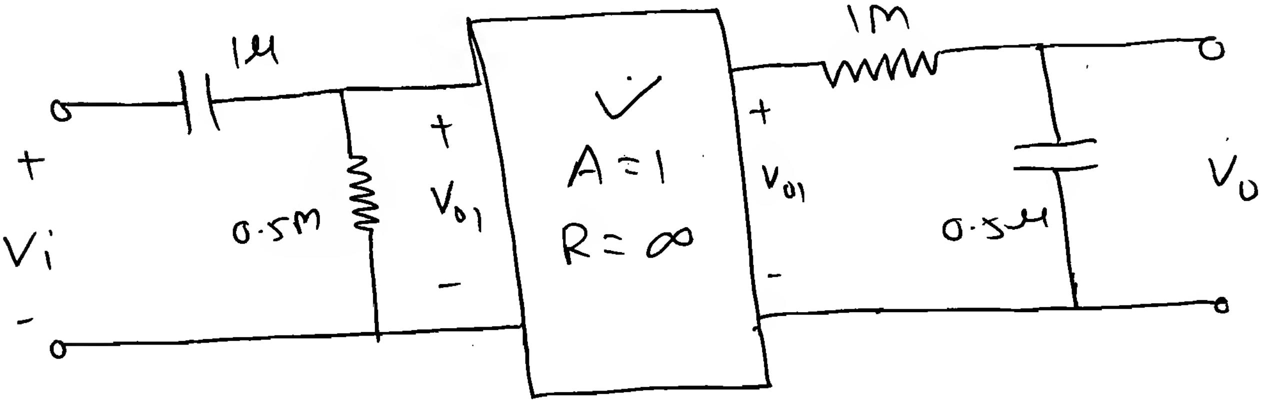
$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{s.1\mu} \times \frac{1}{s.1\mu}}{1M\left(\frac{1}{s.1\mu}+1M+\frac{1}{s.1\mu}\right)+\frac{1}{s.1\mu}\left[1M+\frac{1}{s1\mu}\right]}$$

$$= \frac{\frac{1}{s^2}}{\left(\frac{1}{s}+1+\frac{1}{s}\right)+\left(\frac{1}{s}+\frac{1}{s^2}\right)}$$

$$= \frac{1}{(2+s)\,(s+1)}$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+3s+1}}$$

[Q] Find the TF.



Sol^n:

$$\frac{V_o(s)}{V_i(s)} = \frac{V_o(s)}{V_{01}(s)} \times \frac{V_{01}(s)}{V_i(s)}$$

$$\Rightarrow \frac{V_{01}}{V_i} = \frac{R}{R+\frac{1}{sC}} = \frac{RsC}{1+RsC} = \frac{0.5s}{1+0.5s} = \frac{s}{s+2}$$

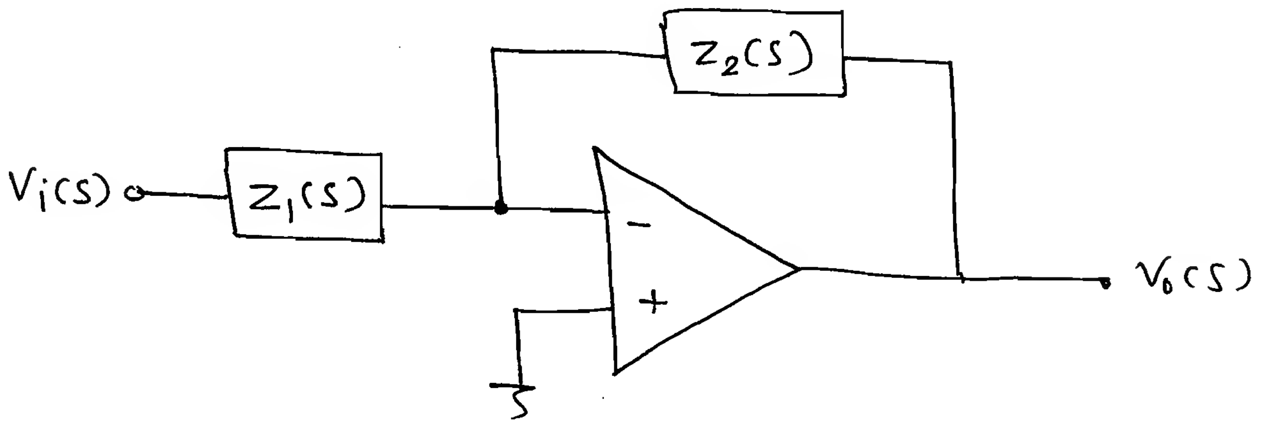
$$\Rightarrow \frac{V_o(s)}{V_{o1}(s)} = \frac{1}{1+sCR} = \frac{1}{1+s/2} = \frac{2}{s+2}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{s}{(s+2)} \times \frac{2}{(s+2)^2} = \frac{2s}{s^2+4s+4}.$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{2s}{(s+2)^2}}$$

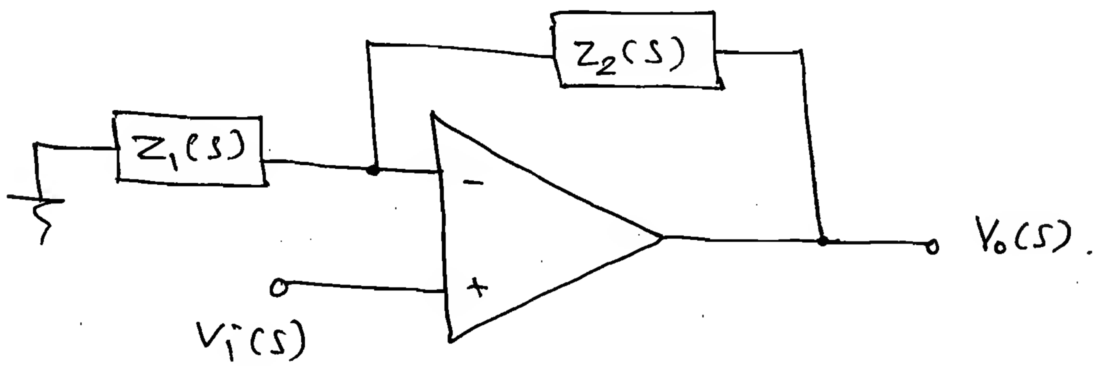
* TF of the OP-AMP :-

① Inverting OP-Amp:



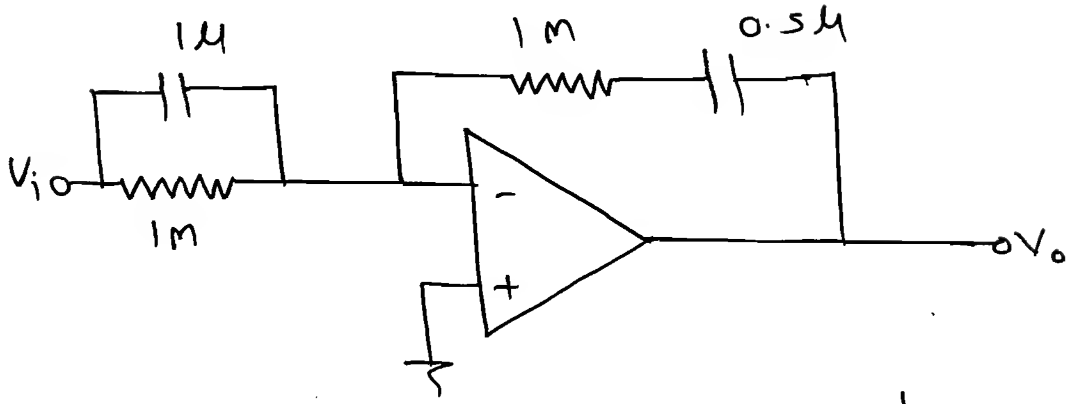
$$\Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}}.$$

② Non-inverting OP-Amp:-



$$\Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)}}$$

[Q] Find the TF.



Sol^n:

$$Z_1(s) = \left(\frac{1}{sC}\right) \parallel (R) = \frac{\frac{1}{sC} \times R}{R + \frac{1}{sC}} = \frac{R}{1 + sCR}$$

$$\rightarrow Z_1(s) = \frac{1m}{1 + s\,1m \cdot 1\mu} = \frac{1m}{s+1}.$$

$$\rightarrow Z_2(s) = R + \frac{1}{sC} = 1m + \frac{1}{0.5\mu s} = 1m + \frac{2}{1\mu s}$$

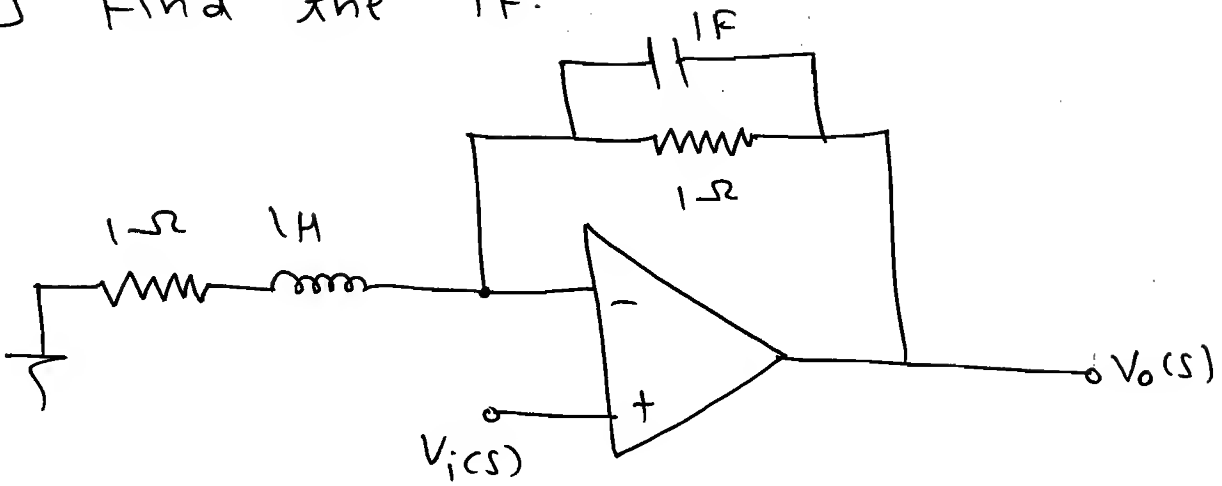
$$= \frac{s+2}{s\,1\mu} = 1m\left(\frac{s+2}{s}\right).$$

$$\therefore \frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = \frac{-1m\left(\frac{s+2}{s}\right)}{1m/(s+1)}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = -\frac{(s+1)(s+2)}{s}$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = -\frac{s^2 + 3s + 2}{s}}$$

a Find the TF.



Sol^n:

$$Z_1(s) = R + sL = (1+s).$$

$$Z_2(s) = R \,||\, \frac{1}{sC} = \frac{R}{1+sCR} = \frac{1}{(s+1)}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)} = 1 + \frac{1}{(s+1)^2}.$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{s^2+2s+2}{s^2+2s+1}}$$

* TF to the Differential equations:-

⇒ Write the TF to the given system where x is ~~output~~ input and y is o/p.

(1) $$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = 2\frac{dx}{dt} + x(t-T).$$

Sol^n: TAKE L.T.

$$\therefore s^3Y(s) + 5s^2Y(s) + 7sY(s) + 9Y(s) = 2sX(s) + e^{-sT}X(s).$$

$$\therefore (s^3 + 5s^2 + 7s + 9)Y(s) = (2s + e^{-sT})X(s).$$

$$\therefore \boxed{TF = \frac{Y(s)}{X(s)} = \frac{2s + e^{-s\tau}}{s^3 + 5s^2 + 7s + 9}}$$

$$\frac{(\text{i/p related term})}{(\text{o/p related term})}$$

(2) $$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + y\frac{dy}{dt} + 10 = \frac{dx}{dt} + x.$$

Soln: The given system is non-linear hence TF is not define. H.B.

[a] Write the differential eqn to the given TF.

$$\frac{Y(s)}{X(s)} = \frac{2s+3}{s^2 + 5s + 6}.$$

Soln:

$$Y(s)(s^2 + 5s + 6) = (2s+3)X(s).$$

$$\Rightarrow s^2Y(s) + 5sY(s) + 6Y(s) = 2sX(s) + 3X(s).$$

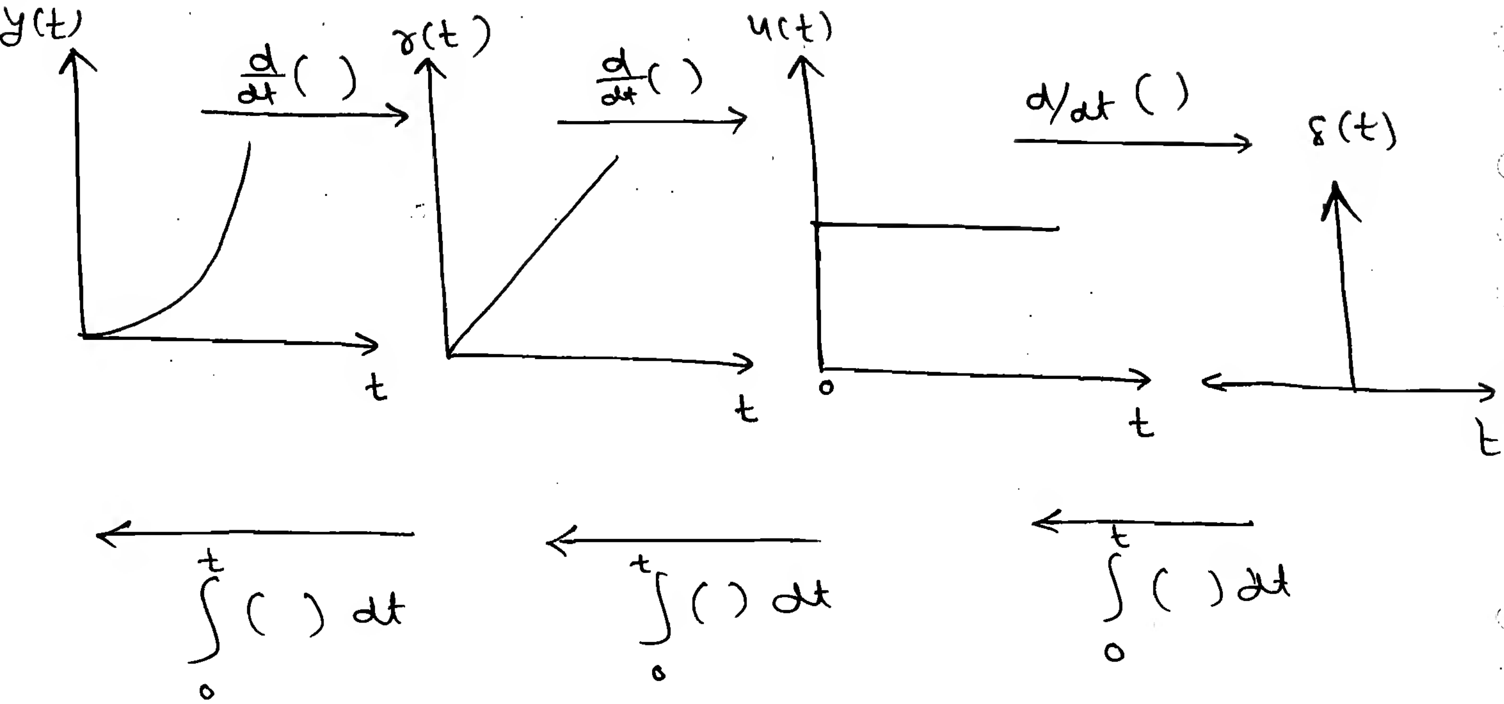
$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 2\frac{dx}{dt} + 3x.}$$

* TF to the signal Response:-

⇒ To get the TF from the signal response used the following formula.

$$\boxed{TF = \left.\frac{L[o/p]}{L[i/p]}\right|_{I_i = 0}}$$

⇒ <u>Conversion</u> <u>of</u> <u>Responses:</u> (1.B.)



[a] The unit step response of the system is

$y(t) = \left(\frac{5}{2} - \frac{5}{2}e^{-2t} + 5t\right)$, $t \geq 0$ its TF is __?

<u>Solⁿ:</u>

$$TF = \frac{L[\text{Unit step Res.}]}{L[\text{Unit step}]}.$$

$$\therefore TF = \frac{\frac{5}{2s} - \frac{5}{2(s+2)} + \frac{5}{s^2}}{1/s}$$

$$= \frac{5s^2(s+2) - 5s^2 + 5(s+2)2}{2(s+2) \times \cancel{s} \cdot s}$$

$$\therefore TF = \frac{10s+10}{2s(s+2)}.$$

$$\therefore \boxed{TF = \frac{5(s+1)}{s(s+2)}.}$$

[Q] The impulse response of the system is $C(t) = (-4e^{-t} + 6e^{-2t})$. $t \geq 0$. The SI equivalent step response is ?

Soln: Step response is $\int_0^t (-4e^{-t} + 6e^{-2t})\,dt$.

$$= \left[4e^{-t} - 3e^{-2t}\right]_0^t$$

$$= \left[4e^{-t} - 3e^{-2t} - 4 + 3\right]$$

$$\boxed{r(t) = 4e^{-t} - 3e^{-2t} - 1.}$$

* Sensitivity:

⇒ The Sensitivity gives the relative variations in the output due to parameter variations in (i) G(s) (ii) H(s).

⇒ Sensitivity of the TF w.r.t. G(s) ⇒ $S_G^T = \dfrac{\%\text{ Change in TF}}{\%\text{ Change in } G}$.

$$\therefore \boxed{S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{G}{T} \times \frac{\partial T}{\partial G}.}$$

Simillarly, $\boxed{S_H^T = \dfrac{H}{T} \times \dfrac{\partial T}{\partial H}.}$

* Find the Sensitivity of the OL and CL sys. w.r.t. Variations. (i) G(s) (ii) H(s).

Soln:

① OL sys.

$$S_G^T = \frac{G}{T} \times \frac{\partial T}{\partial G} = \frac{G}{G} \cdot \frac{\partial G}{\partial G} = 1.$$

② CL sys.

$$T = \frac{G}{1+GH}.$$

$$\Rightarrow \quad S_G^T = \frac{G}{T} \times \frac{\partial T}{\partial G}$$

$$= \frac{\cancel{G}}{\cancel{G}} \times \cancel{(1+GH)} \times \frac{(1+GH)(1) - (G)(H)}{(1+GH)^{\cancel{2}}}$$

$$\therefore \boxed{S_G^T = \frac{1}{1+GH}} \leftarrow \text{H.B}$$

$$\Rightarrow \quad S_H^T = \frac{H}{T} \times \frac{\partial T}{\partial H}$$

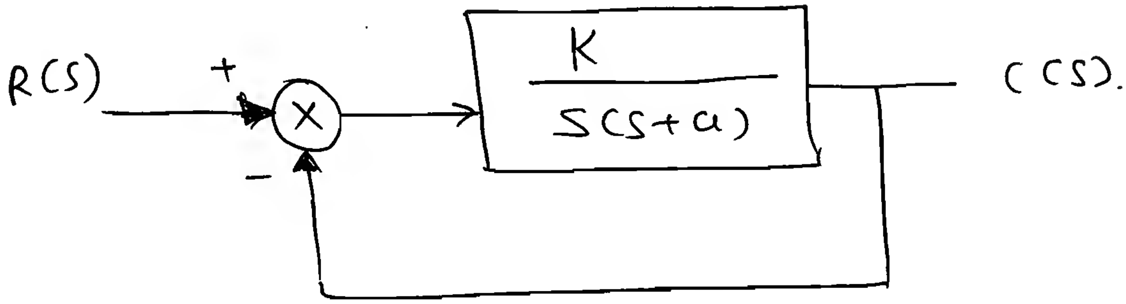
$$= \frac{H}{\cancel{G}} \times \cancel{(1+GH)} \times \frac{-G^{\cancel{2}}}{(1+GH)^{\cancel{2}}}$$

$$\therefore \boxed{S_H^T = \frac{-GH}{(1+GH)}} \checkmark \text{ H.B.}$$

$$\Rightarrow \quad \boxed{S_H^T > S_G^T}$$

⇒ feedback is more sensitive than the forward path.

Q. Find the sensitivity of the system w.r.t. variations in ① K ② a



Soln:

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+a)+K} = \frac{K}{s^2+sa+K}$$

(i) $$S_K^T = \frac{K}{T} \times \frac{\partial T}{\partial K}$$

$$= \frac{\cancel{K}}{\cancel{K}} \times \cancel{(s^2+sa+K)} \times \frac{(s^2+sa+K)(1) - (K)(1)}{(s^2+sa+K)^{\cancel{2}}}$$

$$\boxed{S_K^T = \frac{s^2+as}{s^2+as+K}}$$

(ii) $$S_a^T = \frac{a}{T} \times \frac{\partial T}{\partial a}$$

$$S_a^T = \frac{a}{\cancel{K}} \times \cancel{(s^2+as+K)} \times \frac{-\cancel{K} \times s}{(s^2+sa+K)^2}$$

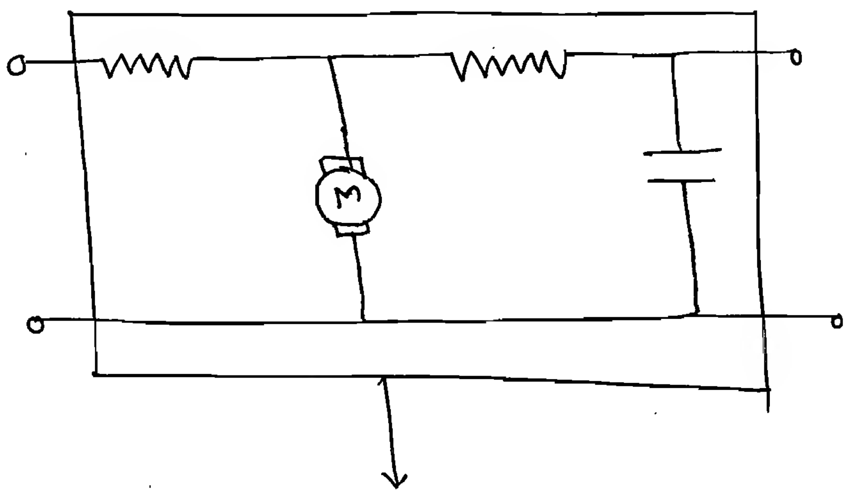
$$\therefore \boxed{S_a^T = \frac{-as}{(s^2+as+K)^2}}$$

BLOCK DIAGRAM :-

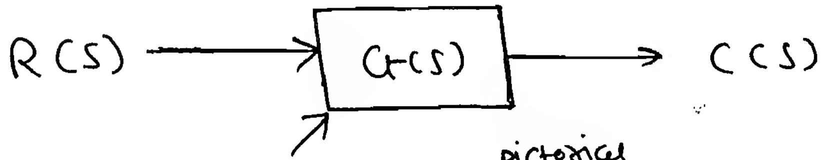
⇒ The purpose of the Block diagram is to find the overall TF of the system.

⇒ A Block Diagram is nothing but the short hand pictorical representation of the system betn input and output.

⇒



⇒



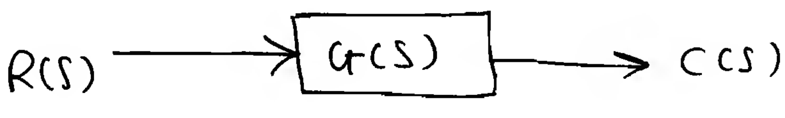
Short-hand + pictorical representation of the sys.

⇒ The systems can be represented in a two ways

① open Loop form.

② Closed Loop form.

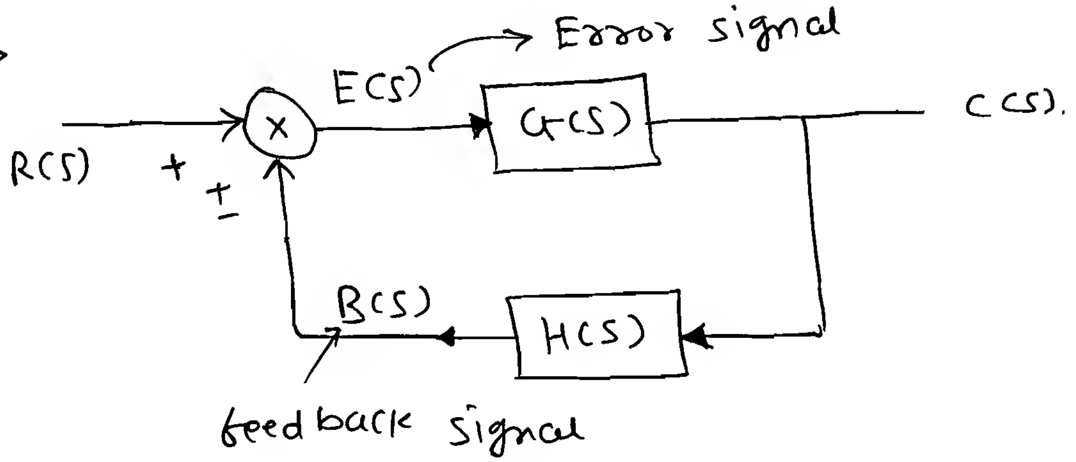
① Open Loop form:



$$\therefore \frac{C(s)}{R(s)} = G(s).$$

② **Closed Loop form:**

⇒



⇒ $G(s)$: Forward path gain $= \dfrac{C(s)}{E(s)}$.

⇒ $H(s)$: Feedback path gain $= \dfrac{B(s)}{C(s)}$.

⇒ $G(s) \cdot H(s)$: Loop Gain (open loop gain).

⇒ $\longrightarrow \boxed{G(s)} \longrightarrow \boxed{H(s)} \longrightarrow B(s)$.

$G(s) \cdot H(s)$ ⇒ OLTF of a Non-unity F/B system.

⇒ $\xrightarrow{H(s)=1} G(s)$ ⇒ OLTF of a Unity FB sys.

⇒ The factor $G(s) \cdot H(s)$ represent the actual closed loop system. It is also called as loop gain (open loop gain).

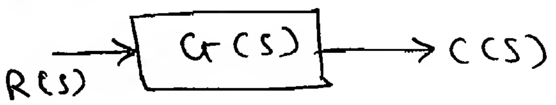
⇒ $$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s) \cdot H(s)}} \rightarrow \text{CLTF}$$

⇒ In a Practical system the phase shift betⁿ feedback signal and input signal is 0° (OR) ±360° whereas for -ve feedback the Phase shift betⁿ i/p and feedback signal is ±180° (OR) out of phase.

* Comparision b/w open Loop system & Closed Loop System.

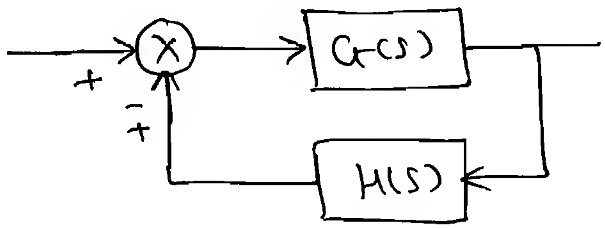
⇒ Open Loop system

*** Gain**



$$G(s) = \frac{C(s)}{R(s)}.$$

Closed Loop system.



→ The main disadvantage of F/B is the gain is reduced by the factor of $\frac{\cancel{G(s)}\ 1}{1+G(s)\cdot H(s)}$.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}.$$

* Stability:

→ Stability is a notion that describes whether the system will be able to follow the input command.

⇒ The OL system is more stable.

→ The CL sys. stability depends on the loop gain.

→ If GH = -1 then the sys. CL sys. stability affected.

→ If GH = 0 then CL sys stability = OL sys. stability.

→ If GH > 0 then the CL sys. more stable than the OL sys.

* Accuracy.

⇒ The OL sys. accuracy depends on the i/p and process.	⇒ The CL sys. accuracy depends on the F/B N/w ratio.
⇒ The OL sys. is Less accurate.	⇒ If the F/B N/w gives the stable value then the CL sys. becomes highly more accurate than OL sys.

* Sensitivity:

⇒ The OL system is highly sensitive w.r.t. the disturbance, noise and environmental condn. because whenever changes occurs in the system it directly affect the o/p.	⇒ The Closed loop sensitivity decreased by the factor of 1+G(s).H(s). i.e. the changes in o/p due to the disturbance, noise and the environmental condn is very less.

BW:

→ For any Practical system the gain BW Product is Constant.

$$BW \propto \frac{1}{t_r} \approx \frac{0.35}{t_r}.$$

→ With feedback the gain is decreased by the factor of $1+GH$. that means the BW increased by $1+GH$.

→ The large BW gives the very quick response.

→ The CL sys. gives the very quick response compared to the OL sys.

* Reliability:

→ The reliability Completely depends on the no. of discrete components used in the system.

⇒ The open loop sys. is more reliable as it has less no. of Components.

⇒ It is less realiable than OL system.

⇒ In OL system it is not necessary to measure the output.

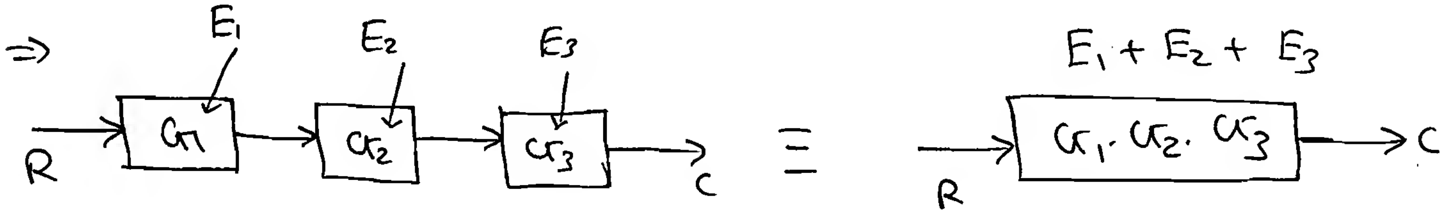
⇒ The output must be measured, error are generated, sensors are

errors are not generated. sensors are not essential design is very easy.	essential and design is complex.

☆ Block Diagram Reduction Techniques:

① Blocks are in series (or) Cascade:-

⇒

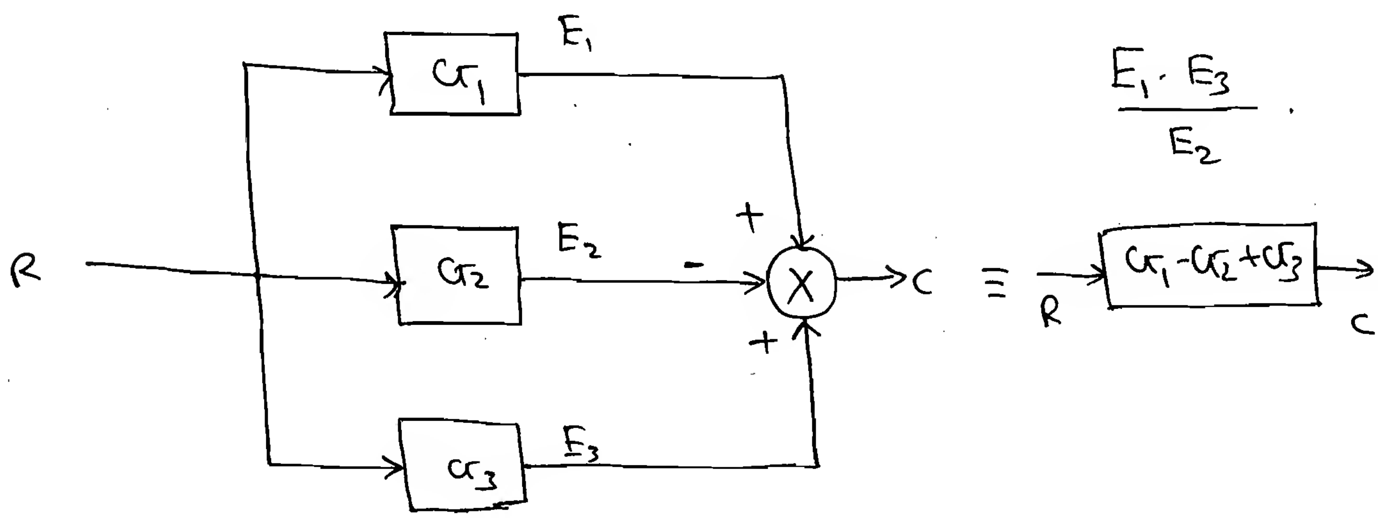


Log

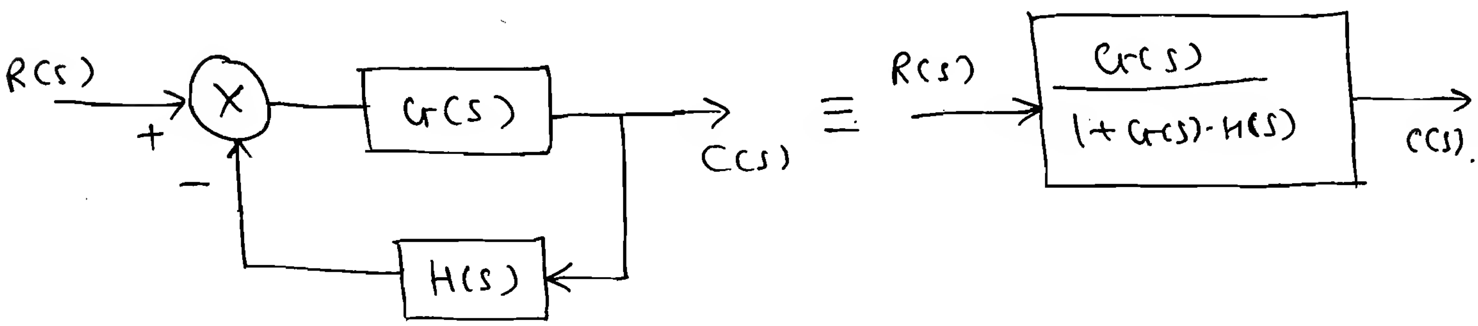
∴ $\times \leftrightarrow +$

$\div \leftrightarrow -$.

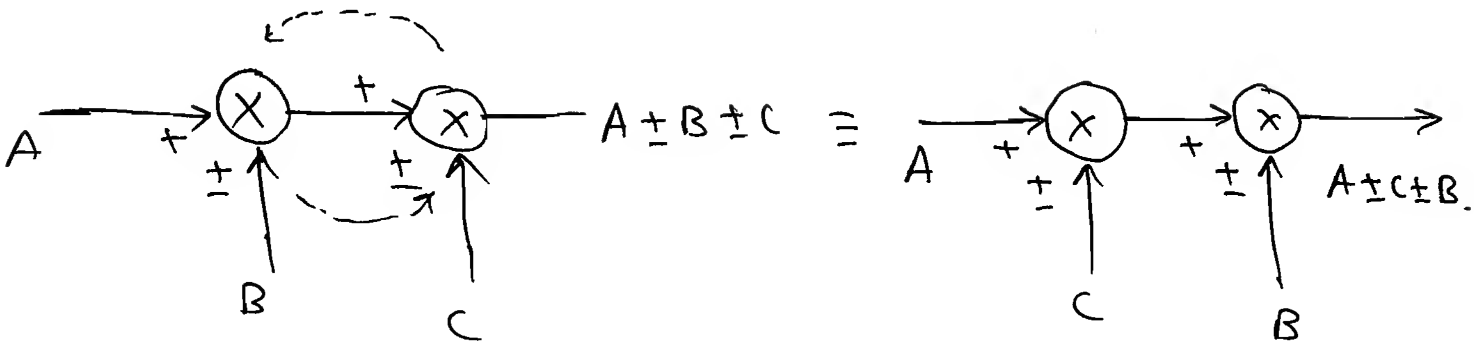
② Blocks are in Parallel:



③ Loop:

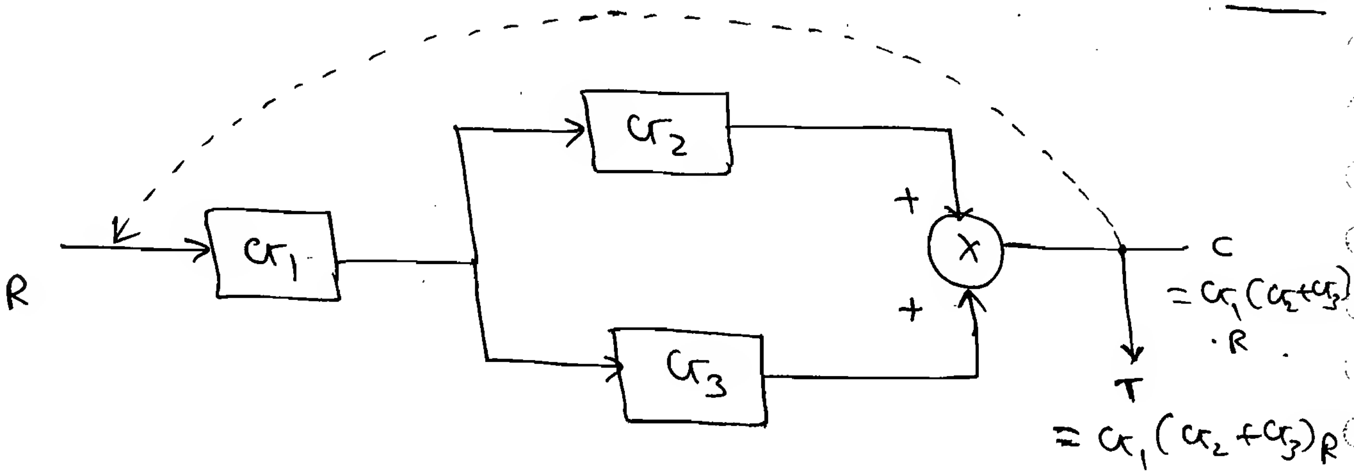


④ Interchanging of Summing Points:

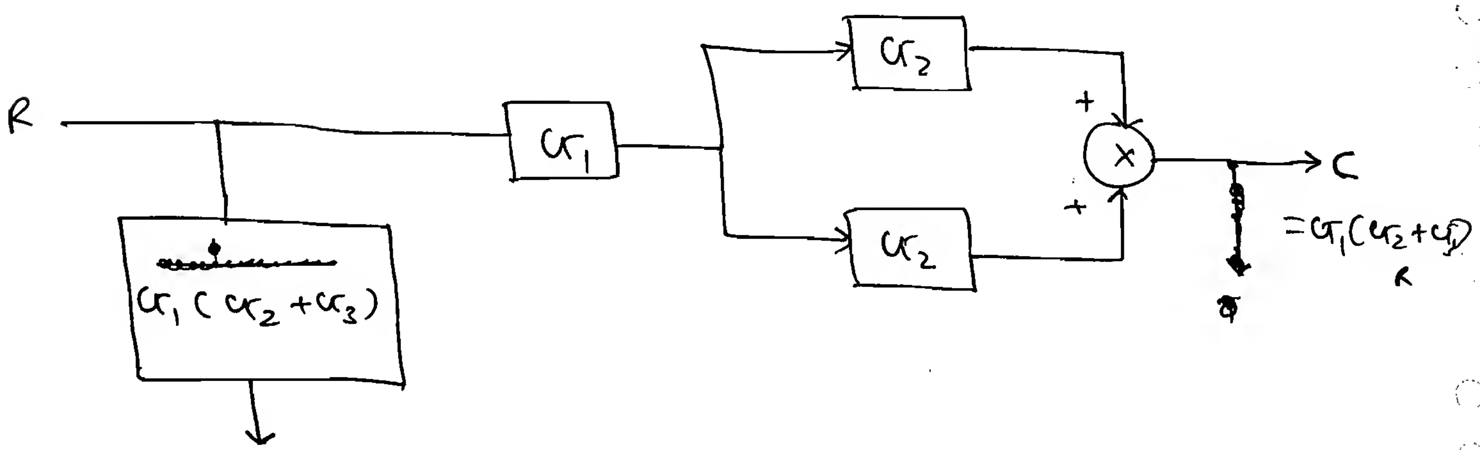


⑤ Adjusting the Block Gain and Take OFF Point.

(i)

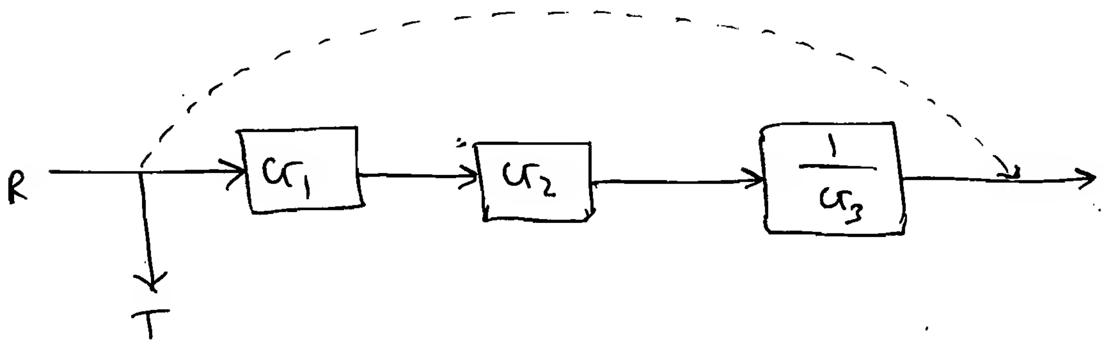


|||



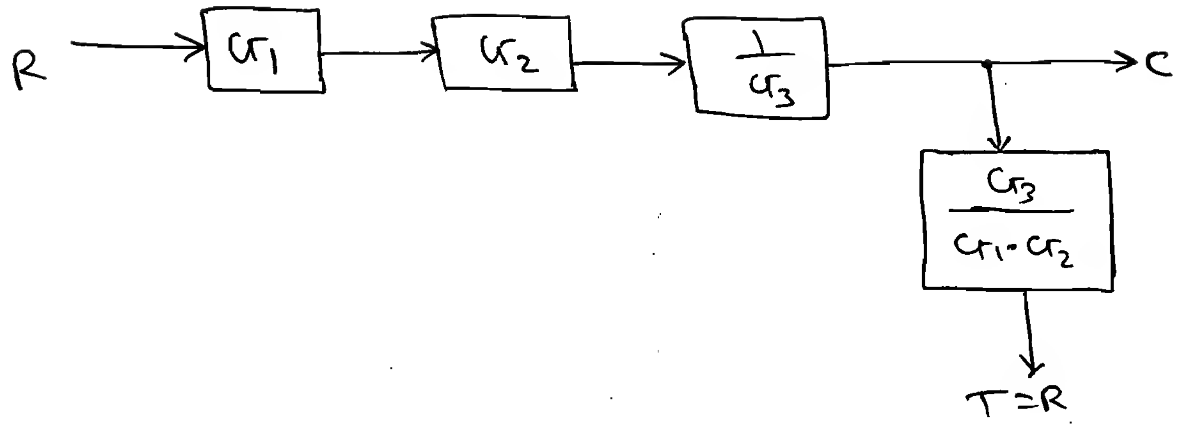
$T = G_1(G_2+G_3)R$

(ii)



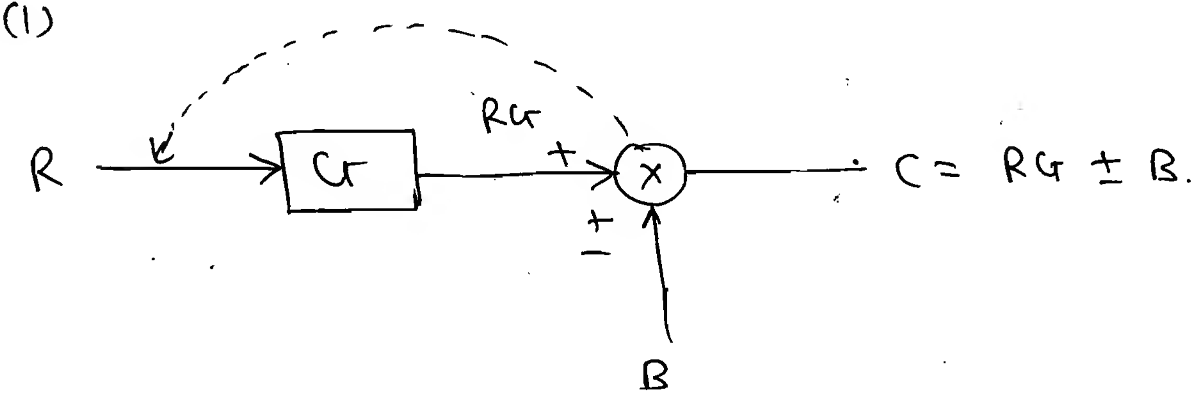
$C = \frac{G_1 \cdot G_2}{G_3}$

|||



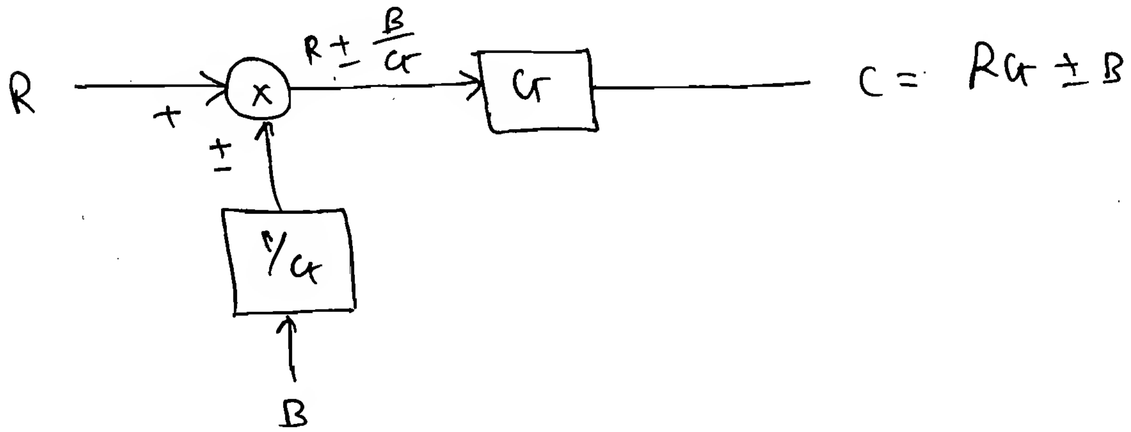
⑥ Adjusting Block Gain & Summing Point.

⇒ (i)

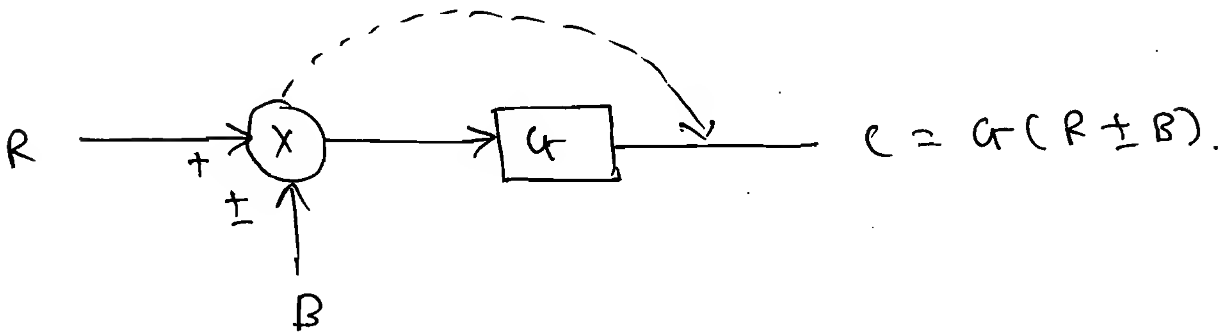


|||

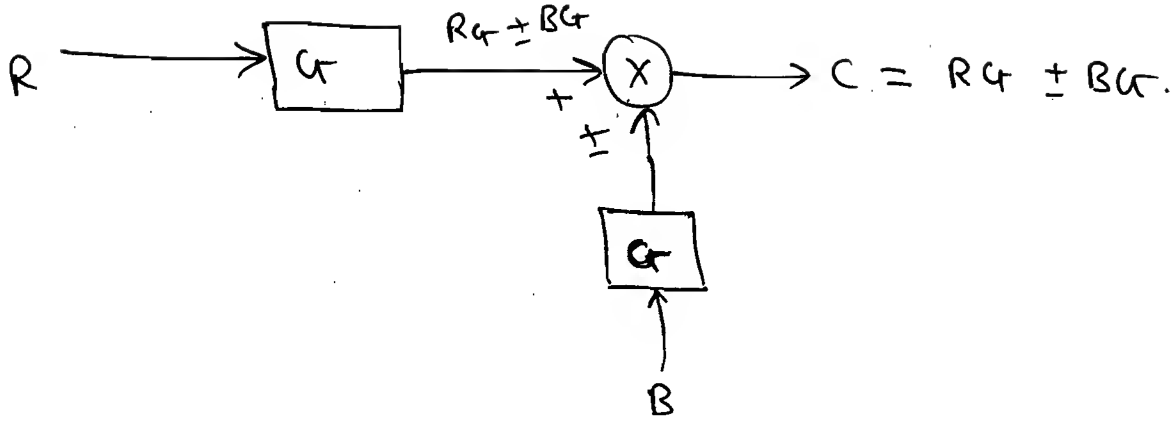
⇒



⇒ (ii)

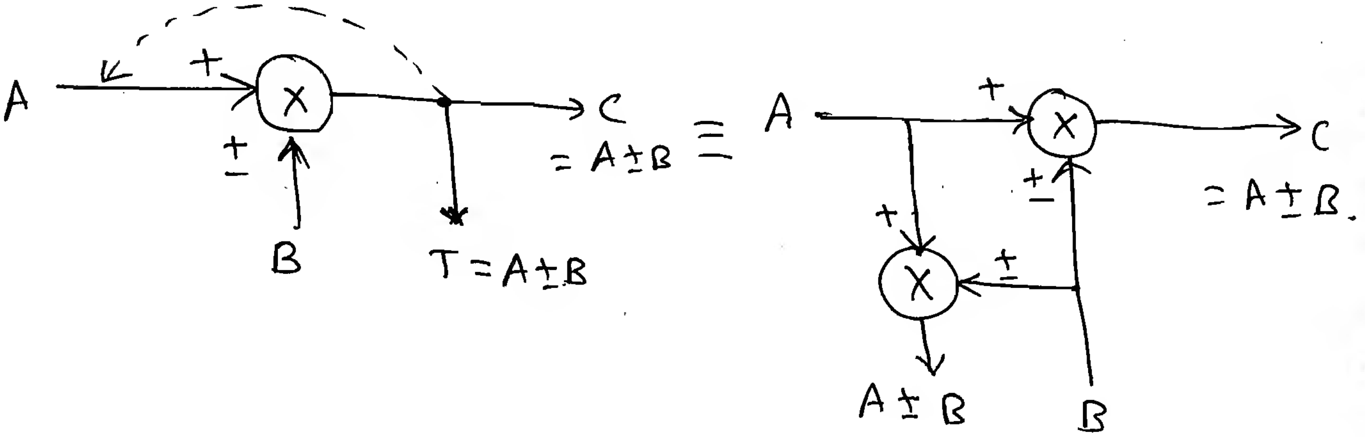


$\Rightarrow$

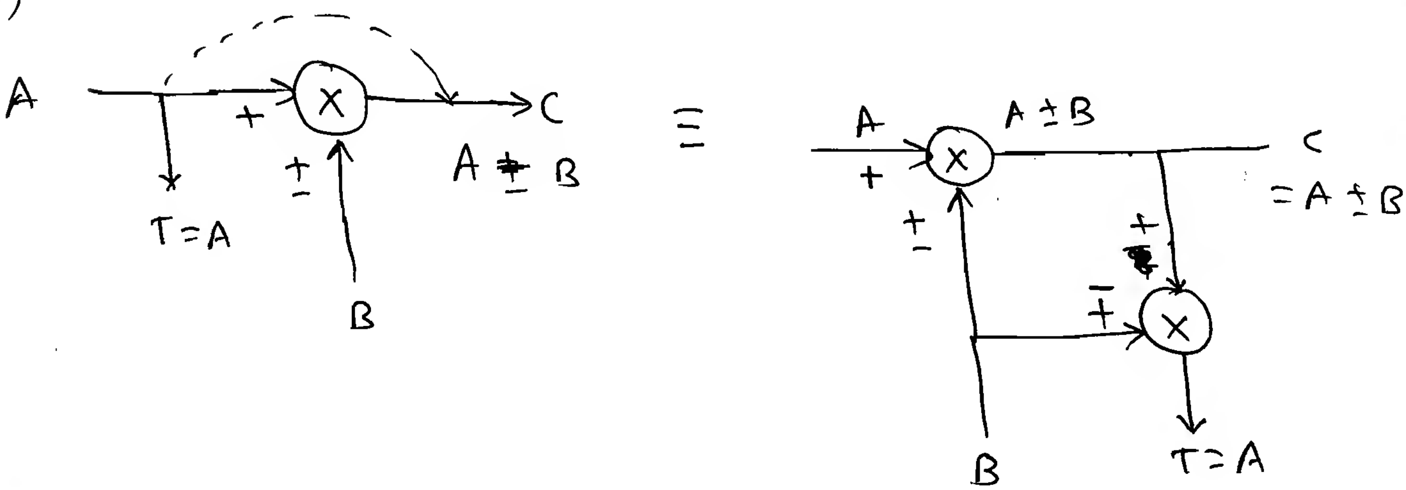


(7) Adjusting the Summing Point & Take OFF point.

(i)



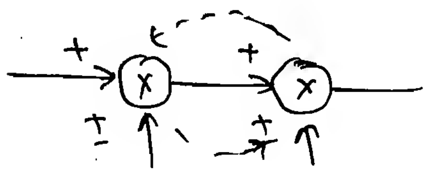
(ii)



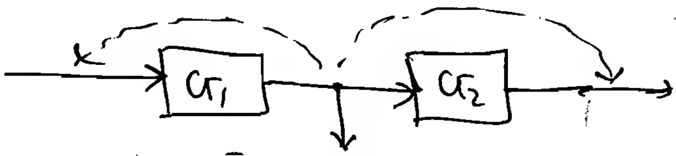
Steps:

(S1) :→ series || llel || Loop.

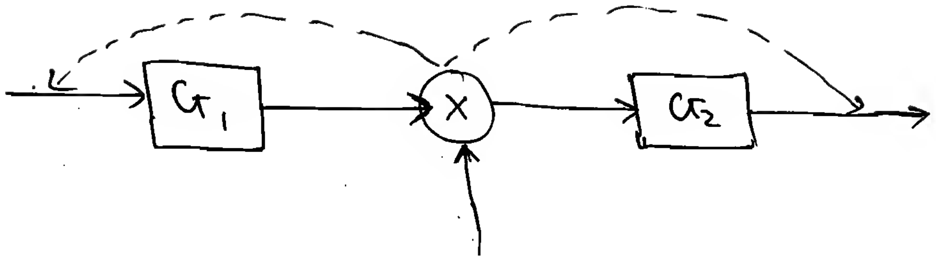
(S2) :→



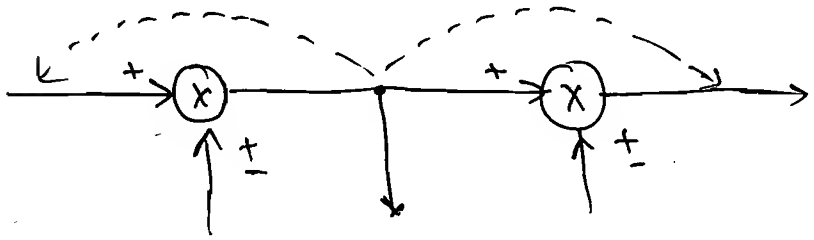
(S3) :→



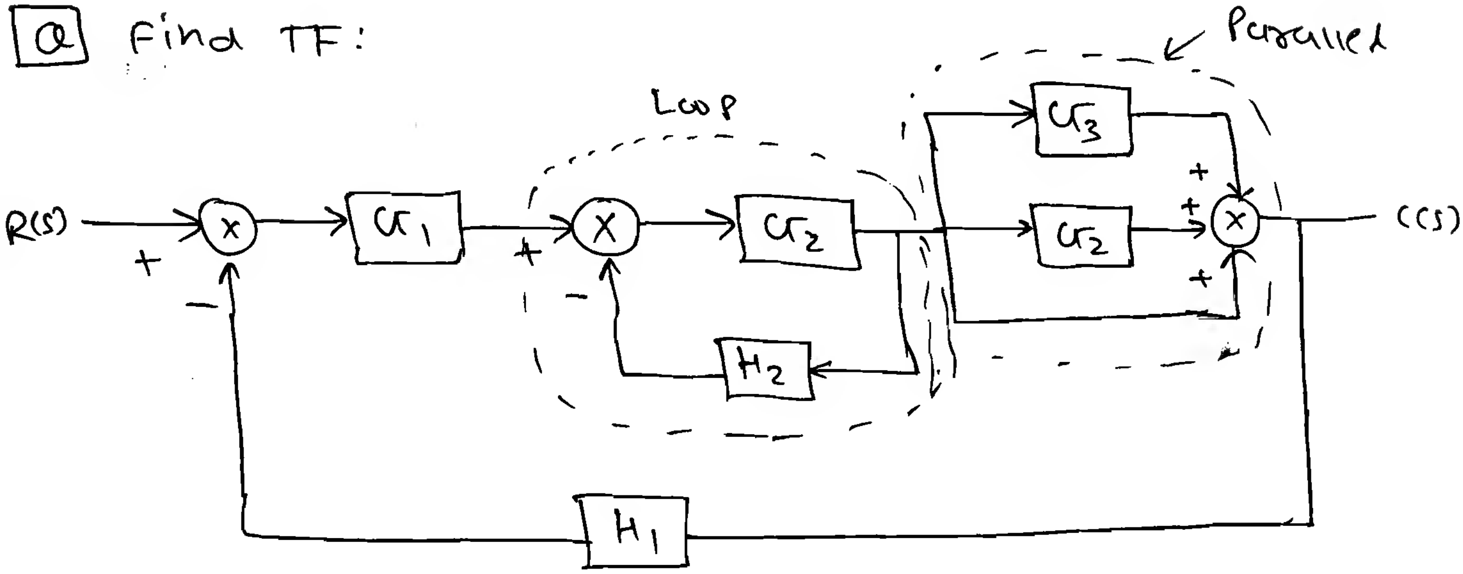
S_4 :



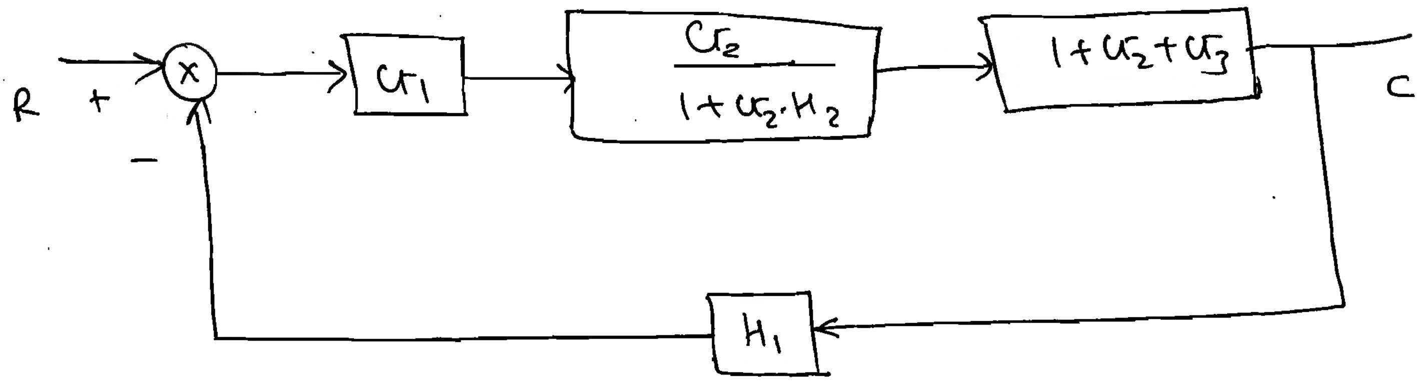
S_5 :

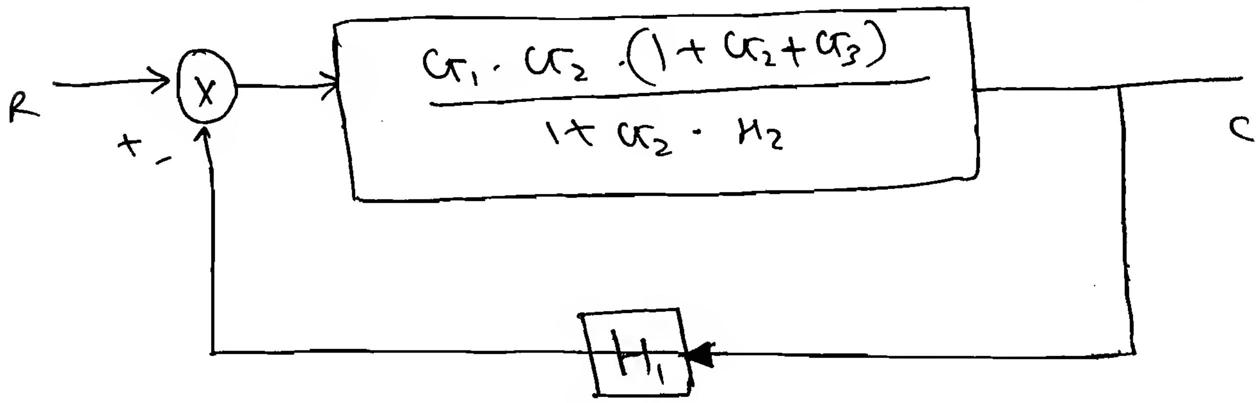


Q Find TF:



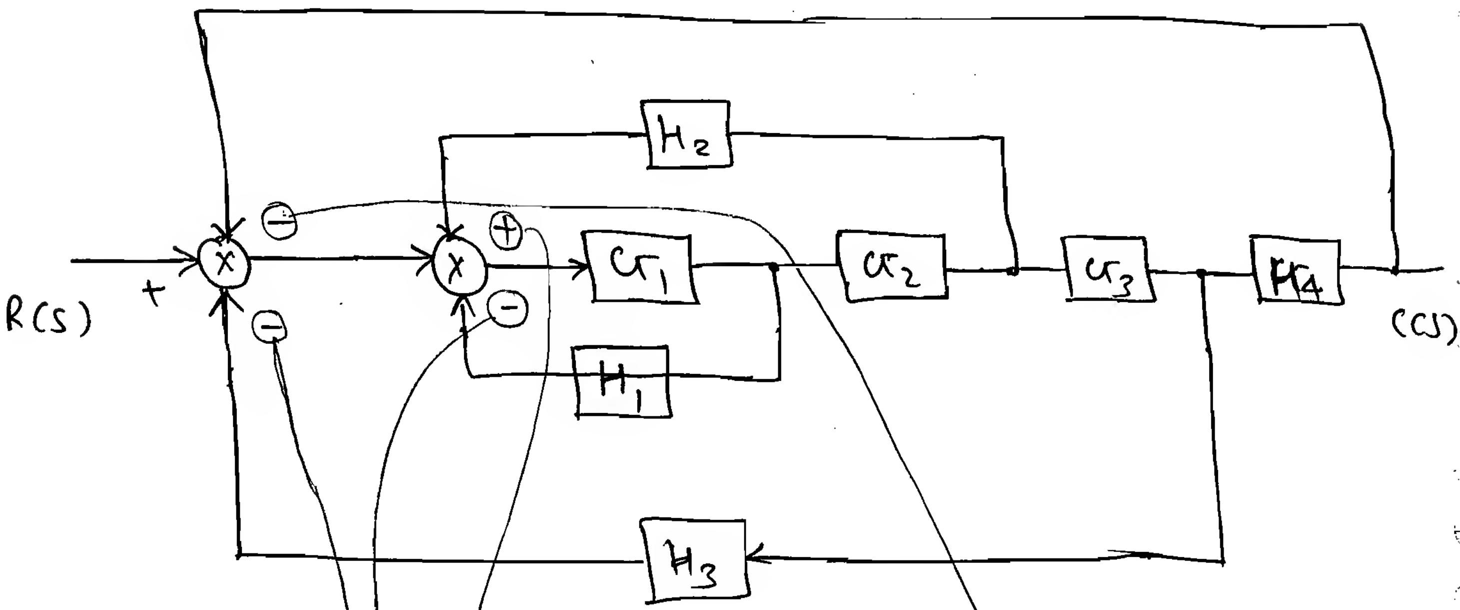
Soln:





$$\Rightarrow \quad \frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \, (1 + G_3 + G_4)}{1 + G_2 H_2 + H_1 \cdot G_1 \cdot G_2 \, (1 + G_3 + G_4)}$$

$\boxed{Q}$ Find TF.

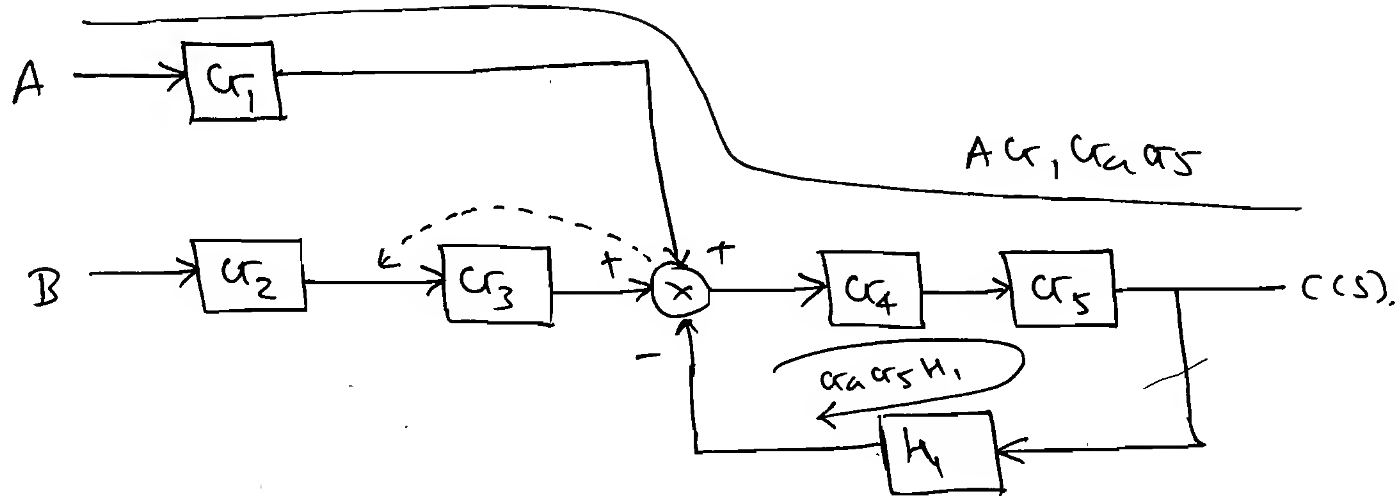


Sol^n:

$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4}{1 + G_1 H_1 - G_1 \cdot G_2 \cdot H_2 + G_1 \cdot G_2 \cdot G_3 \cdot H_4 - G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot 1}$$

Q Draw the eqn Block Diagram to the following. 65

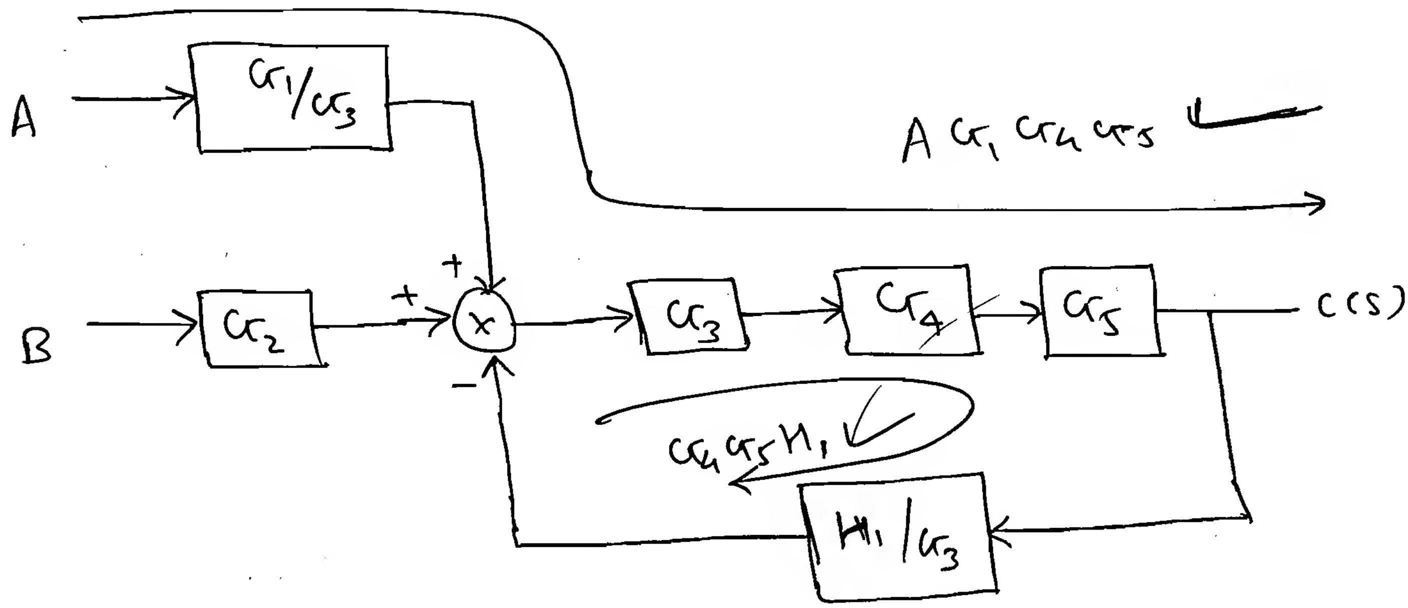
⇒



Soln:

Note: While doing the shifting operation, the
→ changes are occurs only in additional forward path and feedback path connected to that point only. ⇑ (H.B)

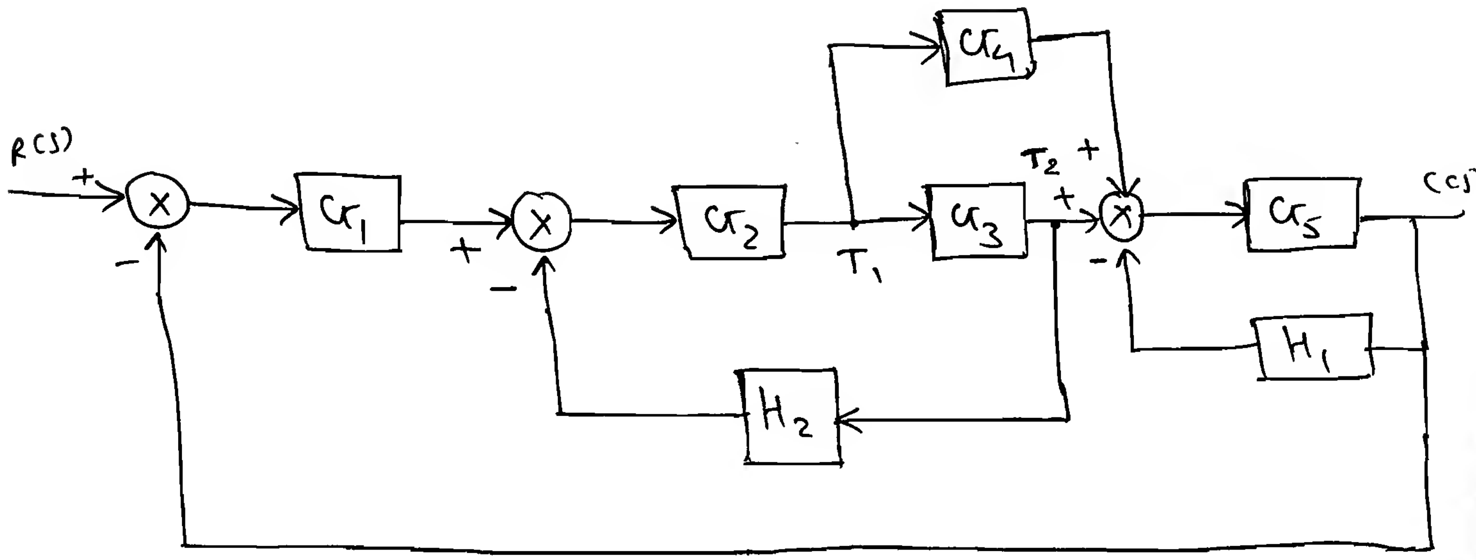
⇒



⇒ Before shifting and after shifting, forward path gain should be remain same. We dont want to loose and we dont want to any

extra gain. So, if it is extra gain after shifting then divide and if it is we loose any gain, we should multiply.

$\boxed{Q}$ Find the TF.



Sol^n: We have 2 option:

(i) Shifting T_1 after G_3.

→ Before shifting there are three block G_1, G_2. & H_2.

→ After shifting there are four block G_1, G_2, G_3, & H_2. So, we should divide G_4 by G_3.

(ii) Shifting T_2 before G_3.

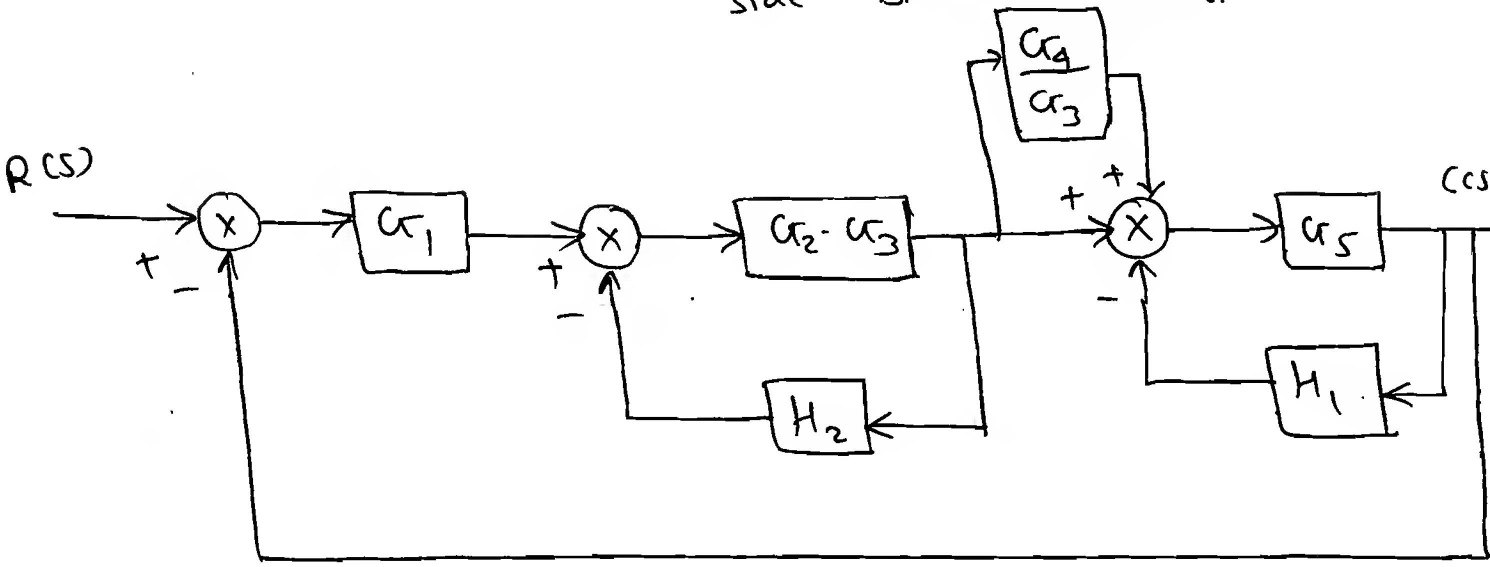
→ Before shifting there are ~~three~~ four blocks G_1, G_2, G_3 & H_2.

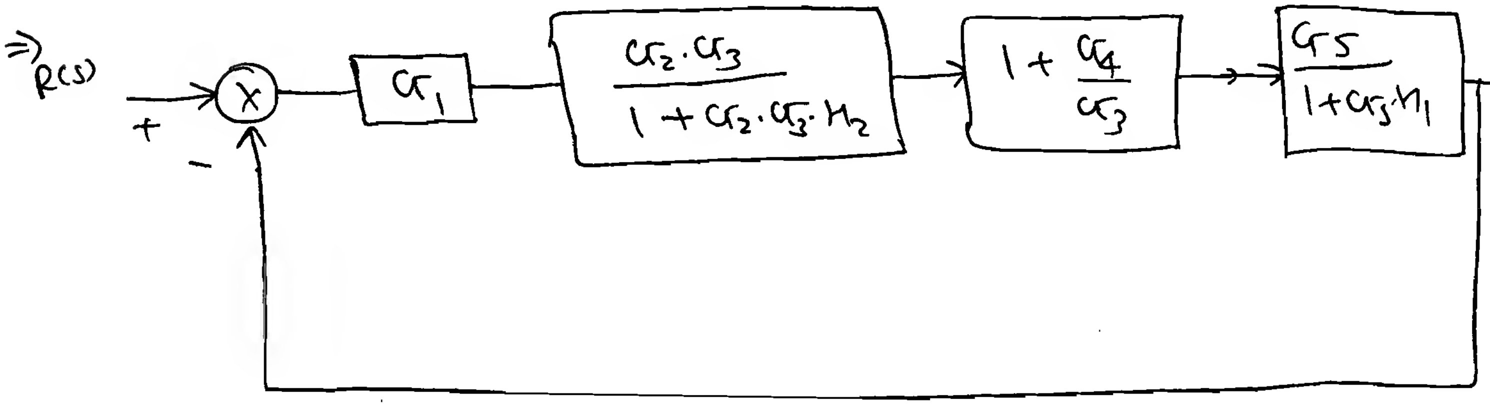
→ After shifting they becomes 3 blocks. i.e. G_1, G_2 & H_2. So, we should mutiply

H_2 by G_3.

Note: For Take off point seek left side block changes. & for summing point see Right side block changes.

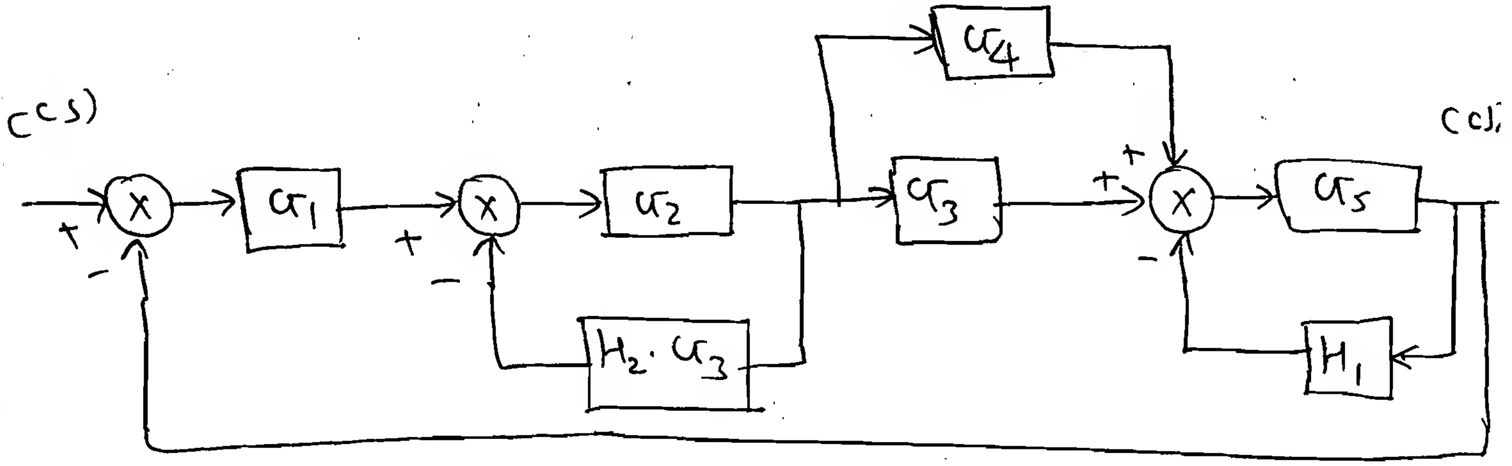
(i) Method - 1:



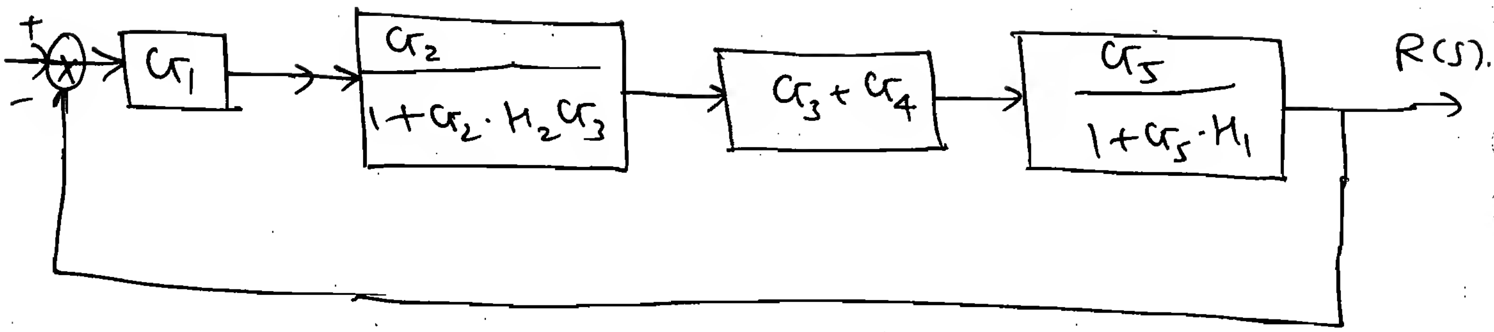


$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \cdot (G_3+G_4) \cdot G_5}{1+G_2 \cdot G_3 \cdot H_2 + G_5 \cdot H_1 + G_1 \cdot G_2 \cdot (G_3+G_4) \cdot G_5}$$

(ii) Method - 2

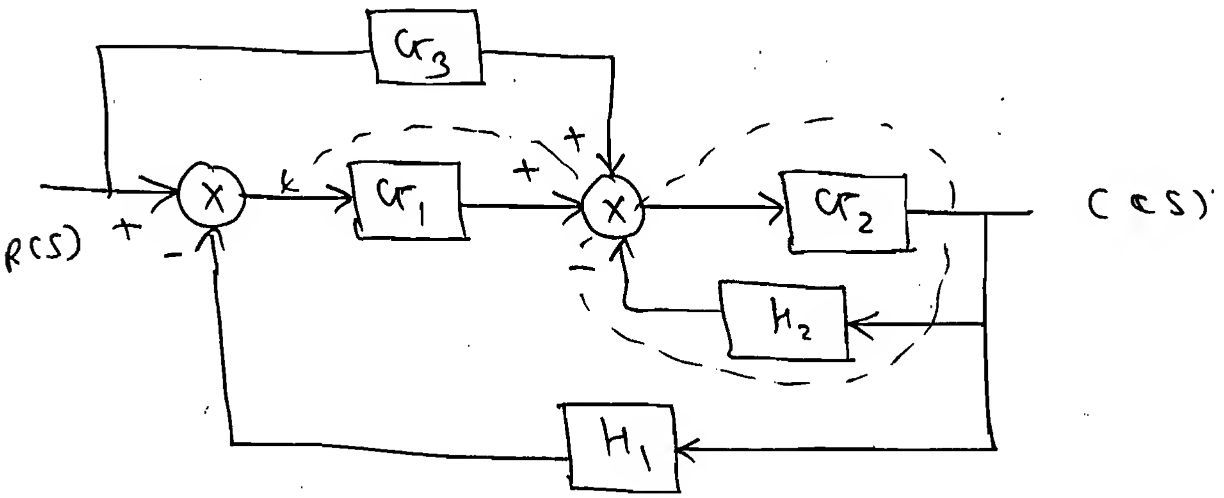


$\Rightarrow$

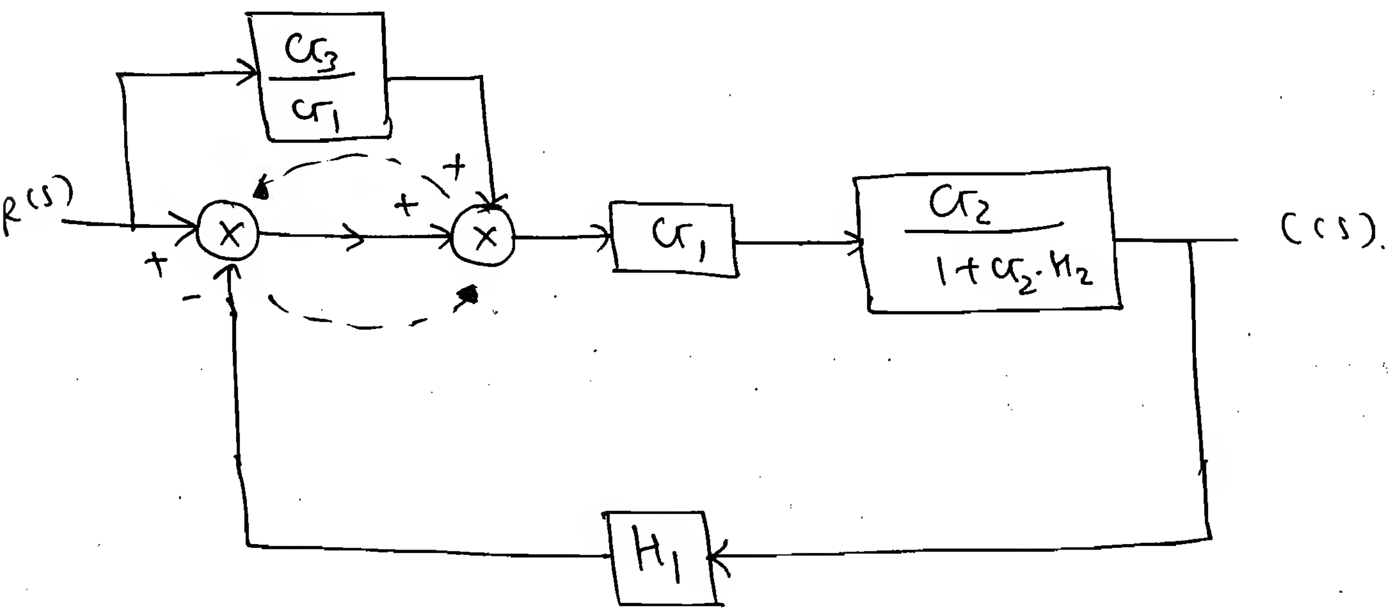


$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \, (G_3+G_4)\, G_5}{1+ G_2 \cdot H_2 \cdot G_3 + G_5 \cdot H_1 + G_1 \cdot G_2 \, (G_3+G_4) \cdot G_5}$$

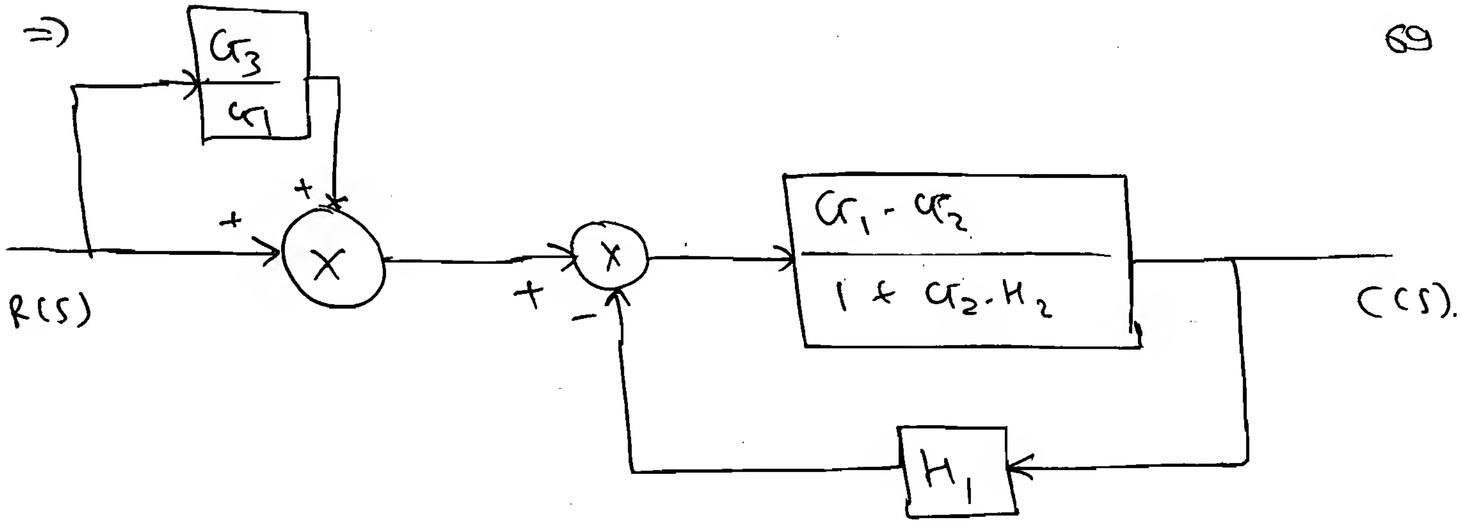
$\boxed{Q}$ Find the TF.



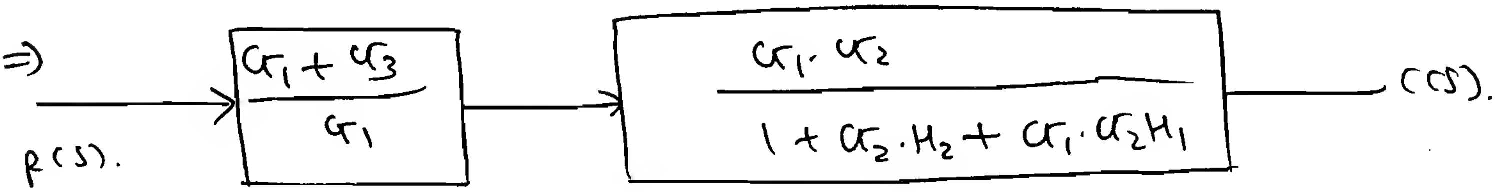
Soln:



$\Rightarrow$

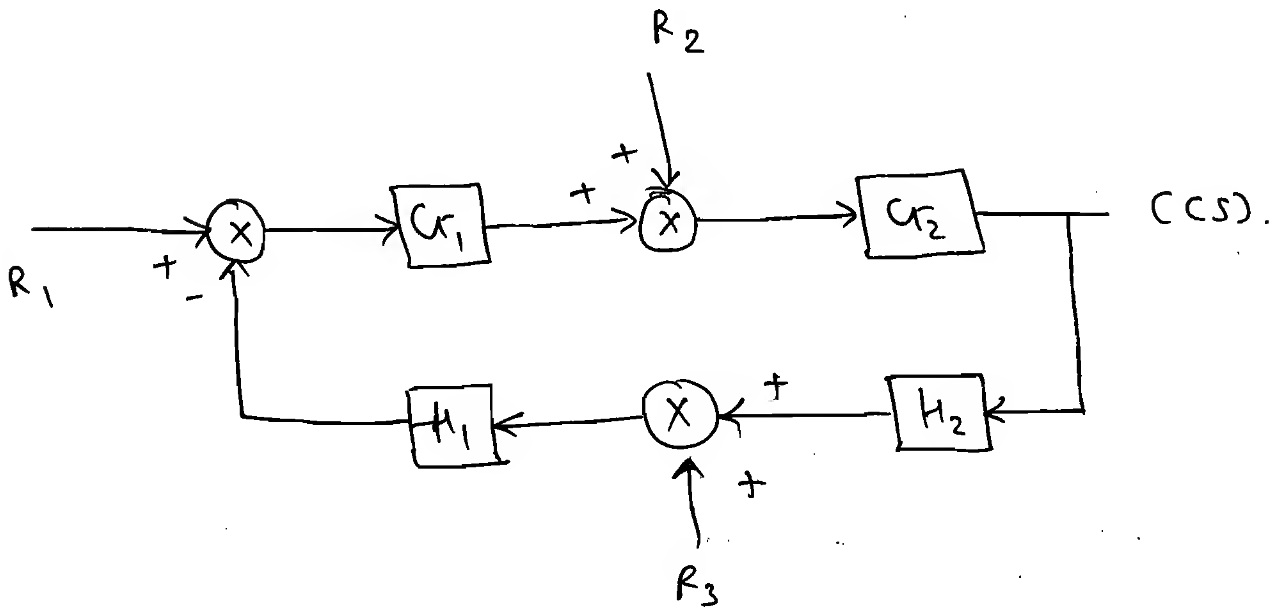


$\Rightarrow$



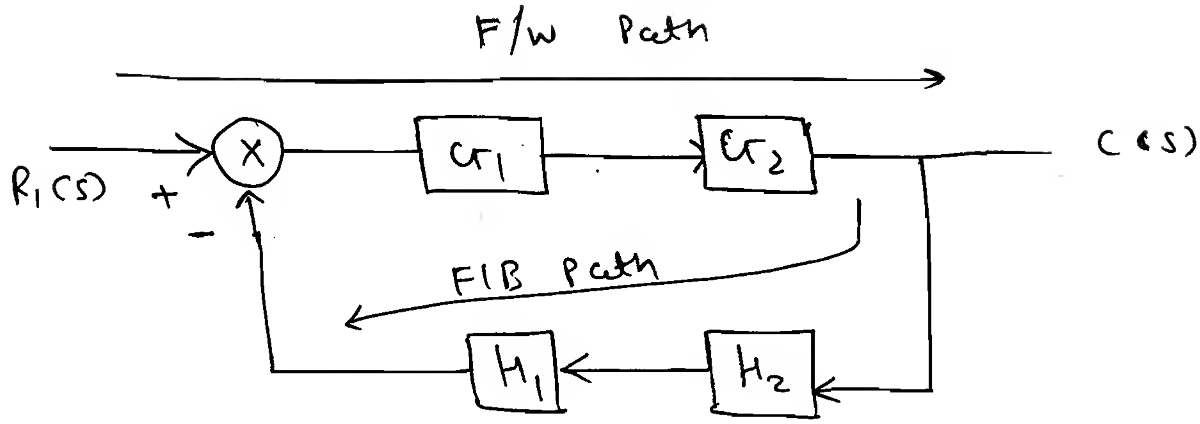
$$\Rightarrow \quad \frac{C(s)}{R(s)} = \frac{G_2 \cdot (G_1 + G_3)}{1 + G_2 \cdot H_2 + G_1 \cdot G_2 H_1}.$$

$\boxed{Q}$ Find the o/p due to the multi i/p.



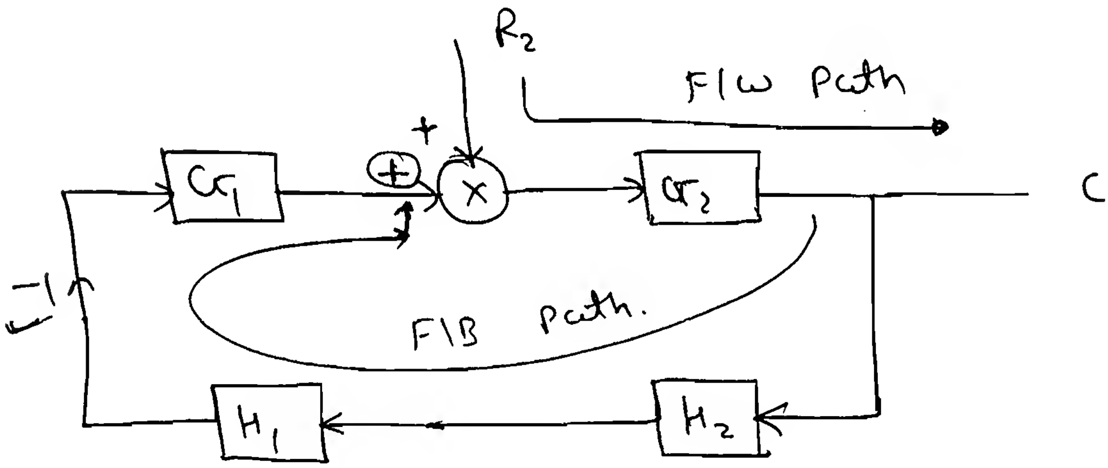
Sol^n: By super possition theorm it can be solved. i.e. take only one input at a time keeping all other zero.

(i) R_1 , $R_2 = 0$, $R_3 = 0$.



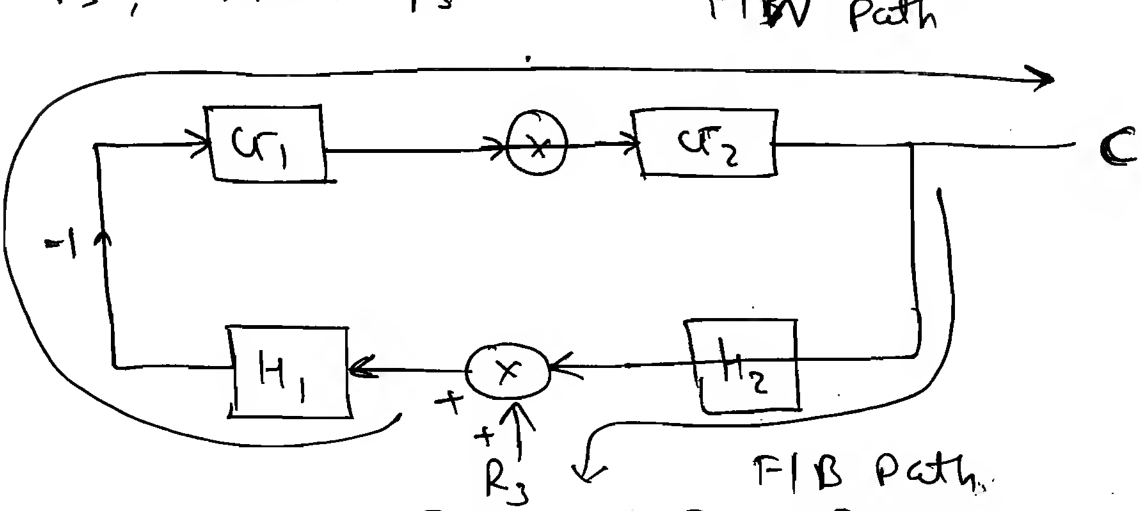
$$\therefore \quad C = \frac{G_1 \cdot G_2}{1 + G_1 \cdot G_2 \cdot H_1 \cdot H_2}.$$

(ii) R_2 , $R_1 = 0$, $R_3 = 0$.



$$\Rightarrow \quad \frac{C}{R_2} = \frac{G_2}{1 - (G_1 \cdot -H_1 \cdot G_2 \cdot H_2)} = \frac{G_2}{1 + G_1 G_2 H_1 H_2}$$

(iii) R_3 , $R_1 = 0$, $R_3 = 0$

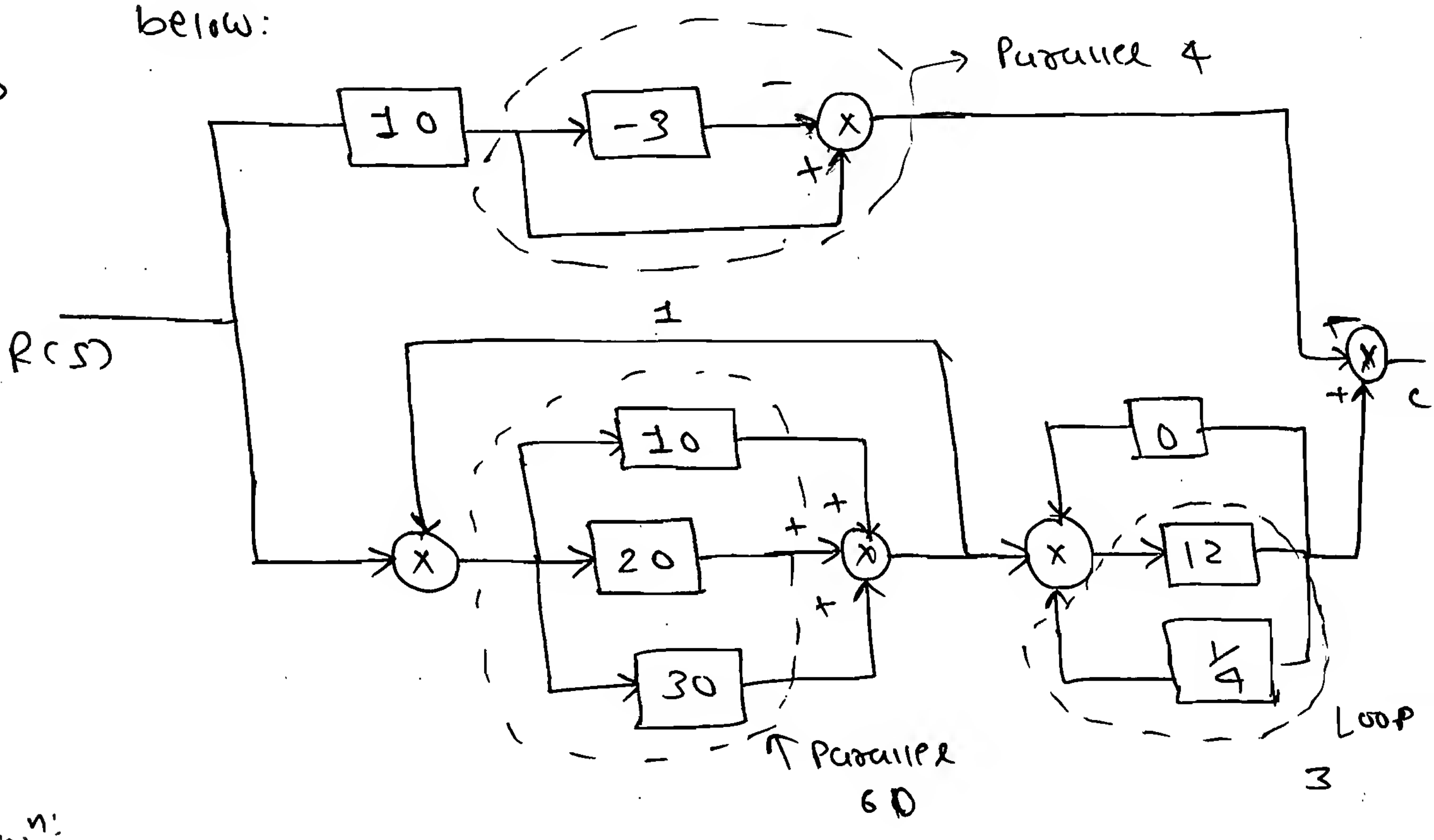


$$\therefore \frac{C}{R_3} = \frac{-H_1 \cdot G_1 \cdot G_2}{1 + G_1 \cdot G_2 \cdot H_1 \cdot H_2}$$

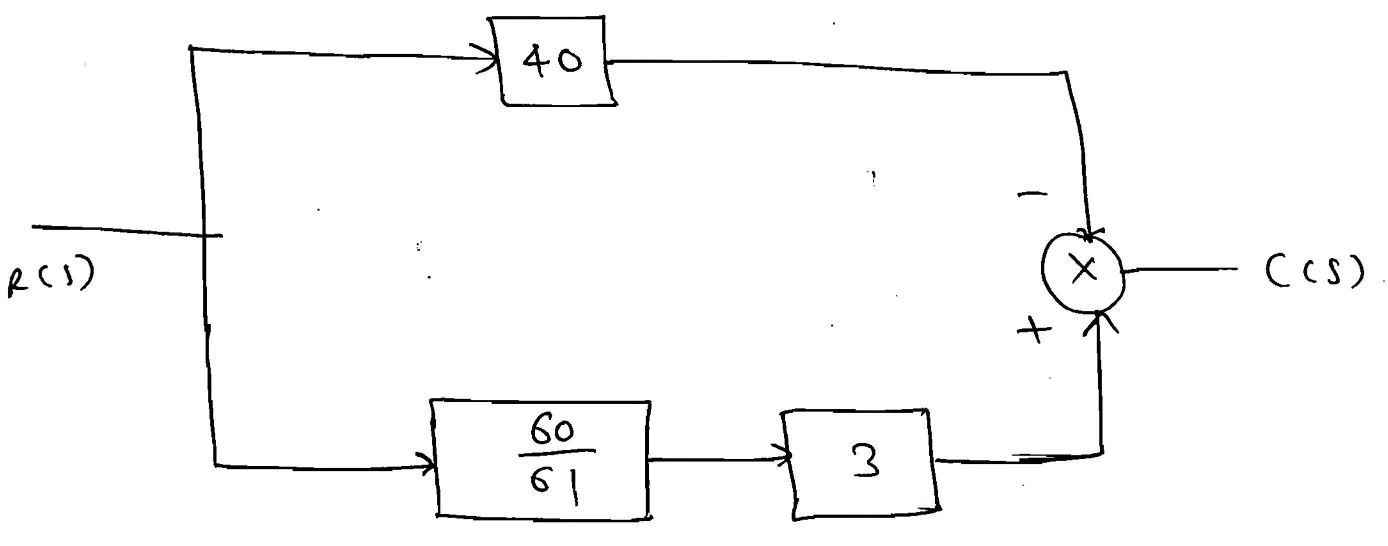
$$\therefore C = \frac{R_1 \cdot G_1 \cdot G_2 + R_2 \cdot G_2 - R_3 \cdot G_1 \cdot G_2 \cdot H_1}{1 + G_1 \cdot G_2 \cdot H_1 \cdot H_2}$$

$\boxed{Q}$ Find the Gain of the system given below:

$\Rightarrow$

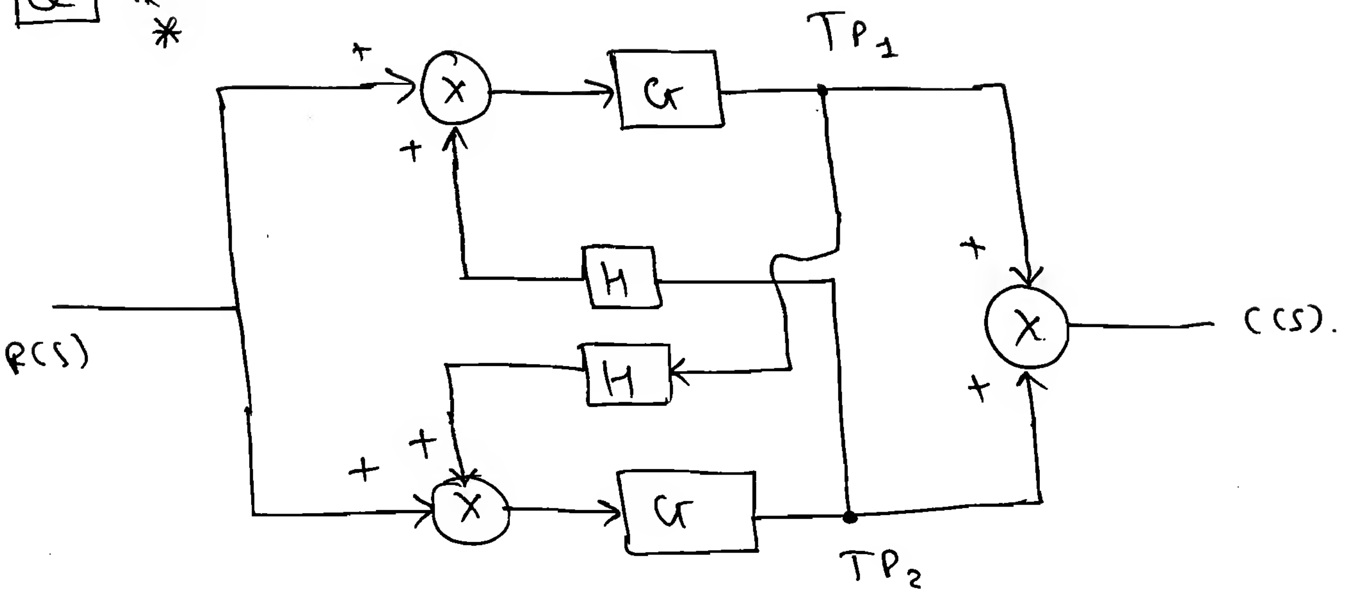


Solⁿ:

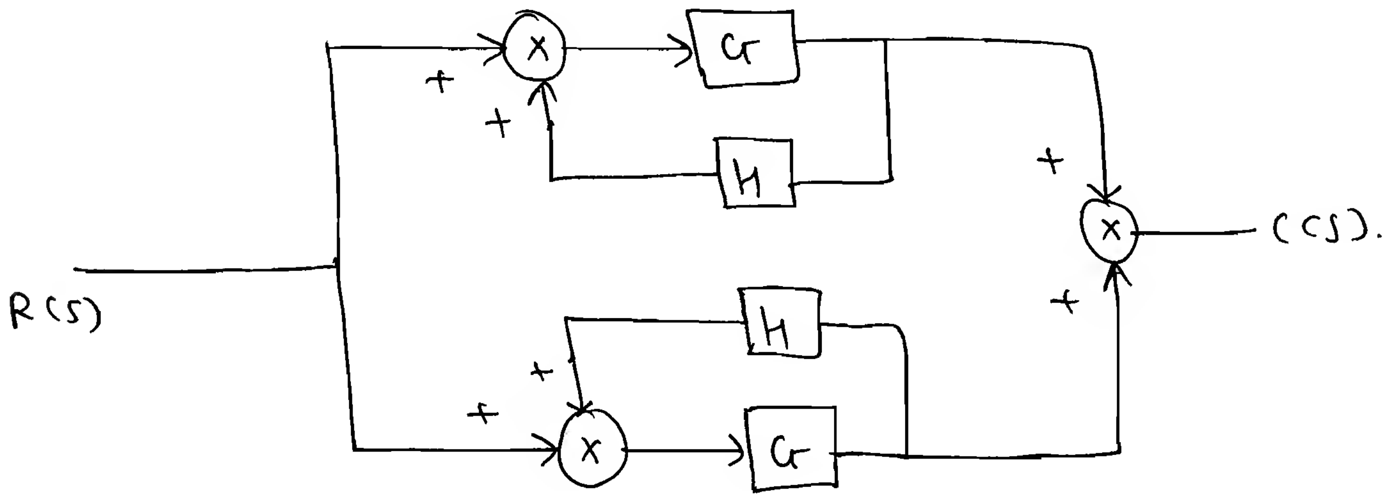


$$\therefore \frac{C(s)}{R(s)} = -40 + 2.95 \simeq -37.$$

Q ***



Soln: In the above example the o/p at TP1 is equal to the o/p at at TP2 at any instant for any i/p. so, they can be interchanged as follow.



So, $$\frac{C(s)}{R(s)} = \frac{G}{1-GH} + \frac{G}{1 \pm GH}.$$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{2G}{1-GH}}$$ //

$\boxed{Q}$ The impulse response of the unity feedback system is

$$c(t) = \left(-t \cdot e^{-t} + 2e^{-t}\right).$$ The open loop TF equal to?

Sol^n: Mention F/B is a CLTF.

$$\therefore \quad \frac{C(s)}{R(s)} = -\frac{1}{(s+1)^2} + \frac{2}{(s+1)}.$$

$\underline{R(s)=1}$ ($\because$ impulse).

$$\therefore \quad \frac{C(s)}{R(s)} = \cancel{+ s+1 + s^2 + 2s + 1}$$

$$= \frac{-1 + 2s + 2}{s^2 + 2s + 1}$$

$$\therefore \quad \frac{G(s)}{1+GH} = \frac{2s+1}{s^2+2s+1}$$

$$\therefore \quad \boxed{G(s) = \frac{2s+1}{s^2}} \leftarrow \text{OLTF}.$$

$\boxed{Q}$ Find the OL DC gain of a unity F/B system of closed loop TF.

$$\frac{C(s)}{R(s)} = \frac{2s+4}{s^2+6s+13}.$$

Sol^n:

$$G(s) = \frac{2s+4}{s^2+4s+9} \leftarrow \text{OLTF}$$

for D.C. $\Rightarrow s=0$.

$\Rightarrow$ OL gain $= 4/9$.

Q The impulse response of a system is $5e^{-2t}$. To produced the responsed of $t \cdot e^{-2t}$. The i/p must be equal to __?

Soln:

$$g(t) = 5 \cdot e^{-2t},$$

$$c(t) = t \cdot e^{-2t}.$$

$$\therefore \quad G(s) = \frac{C(s)}{R(s)}.$$

$$\therefore \quad R(s) = \frac{C(s)}{G(s)}.$$

$$\therefore \quad R(s) = \frac{\frac{1}{(s+2)^2}}{5 \frac{1}{(s+2)}} = \frac{1}{5(s+2)}.$$

$$\therefore \quad \boxed{r(t) = \frac{1}{5} \cdot e^{-2t}}$$

$$\Rightarrow \quad \boxed{r(t) = 0.2 \cdot e^{-2t}}$$

//

☆ Signal Flow Graph (SFG):

* Purpose:

→ To find the overall TF of the system.

→ SFG is the graphical representation of the set of Linear algebric eqⁿs betⁿ i/p and output.

→ The SFG analysis developed to avoid the mathematical calculation like solving the integro, differential eqⁿs (or) Linear algebric eqⁿs.

⇒ The SFG analysis is very easy as compared to solving the mathematical eqⁿs.

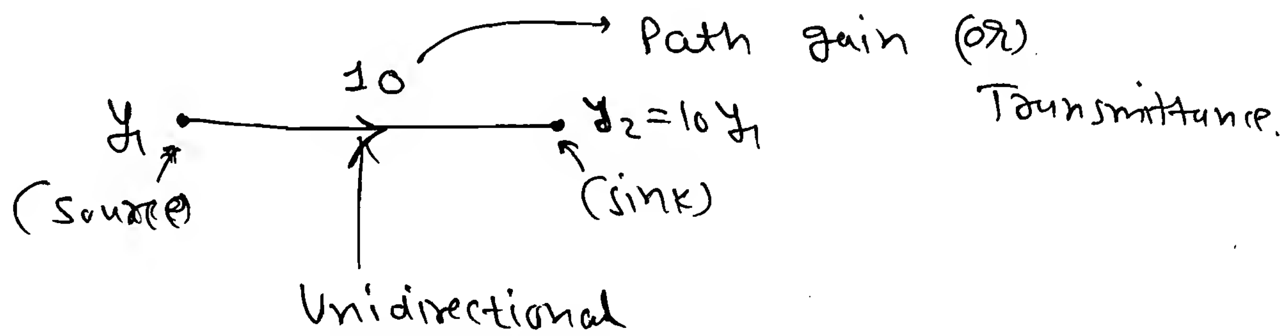
* Construction of SFG to the Linear algebric eqⁿs:

① $y_2 = 10 y_1$

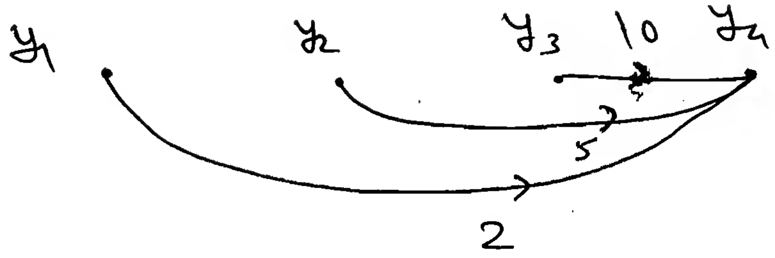
⇒ $y_2 = 10 \cdot y_1$ ← i/p node

o/p node

gain

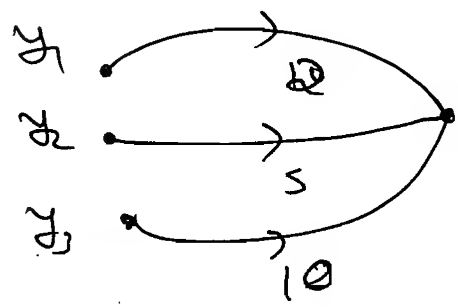
⇒ 

② $y_4 = 2y_1 + 5y_2 + 10y_3$



(many to one)

||

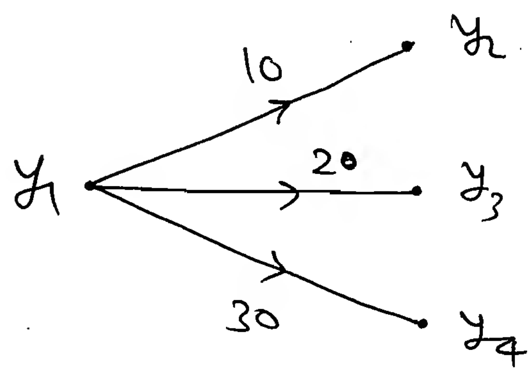


$y_4 = 2y_1 + 5y_2 + 10y_3$

③ $y_2 = 10y_1$.

$y_3 = 20y_1$.

$y_4 = 30y_1$.



(one to many)

* Construct the SFG from the given sets of Linear algebric eqns:

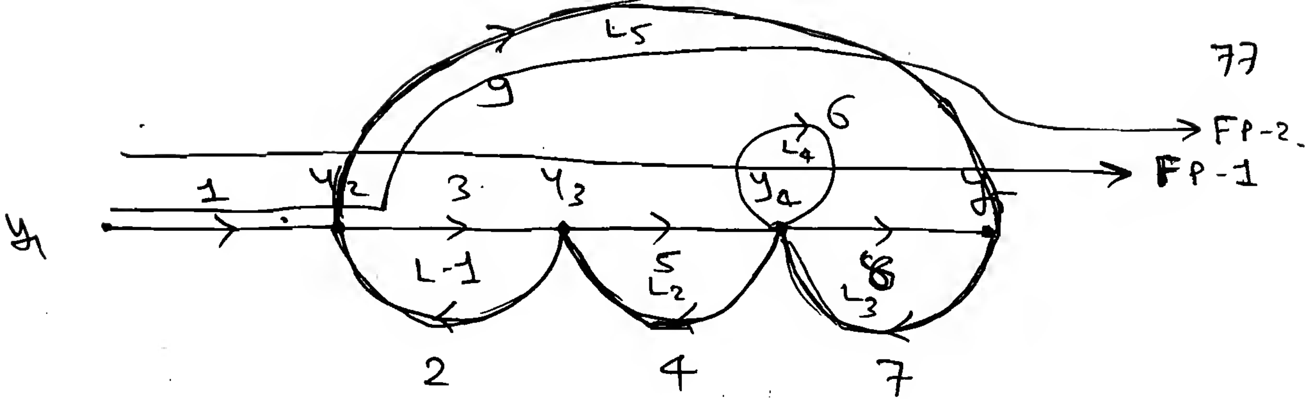
① $y_2 = y_1 + 2y_3$.

$y_3 = 3y_2 + 4y_4$.

$y_4 = 5y_3 + 6y_4 + 7y_5$

$y_5 = 8y_4 + 9y_2$.

Sol^n:



$\boxed{Q-2}$ Find the no. of forward paths, no. of individual loops, no. of two non-touching loops to the above signal graph.

Sol^n: Forward path:

$F_1 \rightarrow$ 1 . 3 . 5 . 8 .

$F_2 \rightarrow$ 1 . 9 .

→ no. of individual loop:

(node)

L_1 : 3 . 2 (2,3) L_4 : 6 (4)

L_2 : 5 . 4 (3,4) L_5 : 9 . 7 . 4 . 2 . (2,3,4,5).

L_3 : 8 . 7 (4,5)

→ Two non-touching loop. (if common node then touching otherwise non-touching).

⊙ $L_1 \rightarrow$ L_2 ✗
L_3 ✓
L_4 ✓
L_5 ✗

⊙ $L_2 \rightarrow$ L_1 ✗
L_3 ✗
L_4 ✗
L_5 ✗

⊙ $L_3 \rightarrow$ L_1 ✓
L_2 ✗
L_4 ✗
L_5 ✗

⊙ $L_4 \rightarrow$ L_1 ✓
L_2 ✗
L_3 ✗
L_5 ✗

⊙ $L_5 \rightarrow$ L_1 ✗
L_2 ✗
L_3 ✗
L_4 ✗

no. 06

So, non- touching loop → 2.

$L_1 L_3$, $L_1 L_4$.

* Loop:

⇒ It is a Path which terminate at the same node where it is started.

* Non- touching Loops:

→ If there is a no common node betn two (OR) more loops then it is said to be the non- touching loop.

* Forward Path:

⇒ It is the Path from input to output.

* Input node:

⇒ A node which has only outgoing branches is called Input node.

* Output node:

⇒ A node which has only incomming branches is called output node.

* Chain (OR) Link node:

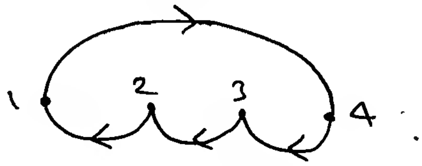
⇒ The node which has both incoming

and outgoing branch.

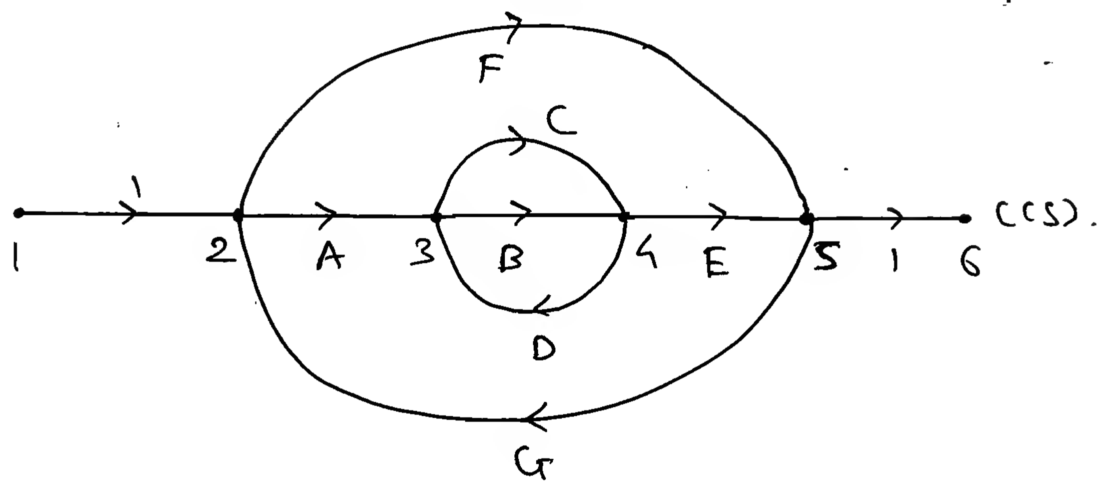
NOTE:

⇒ The Condition to select the Correct Path (or) Loop is each node should be touch only once.

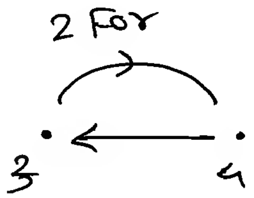
⇒ [Whenever many feedback are cascade with only one forward path it forms a Loop].



Q



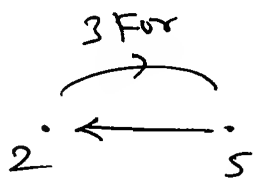
Sol^n:

D ⇒ 

⇒ 2L

$L_1 = BD \rightarrow 3, 4$

$L_2 = CD \Rightarrow 3, 4.$

G ⇒ 

⇒ 3L

$L_3 = ABEG \rightarrow 2, 3, 4, 5$

$L_4 = ACEG \rightarrow 2, 3, 4, 5$

$L_5 = FG \rightarrow 2, 5.$

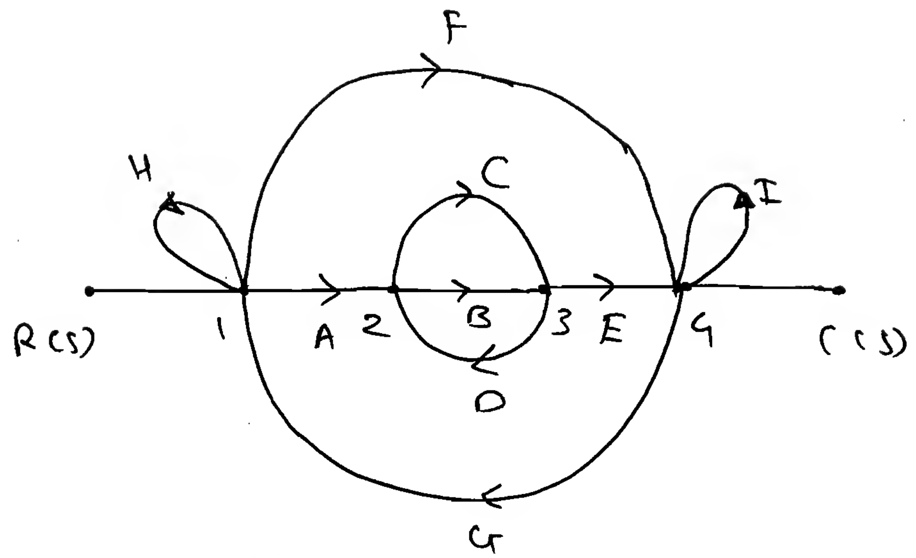
indi → 5L

2 - NTL:

$L_1 L_5$ (3, 4, 2, 5) ✓

$L_2 L_5$ (3, 4, 2, 5) ✓

$\boxed{Q}$



Solⁿ:

D ⟹ 2 For ⟹ (2L)

$\to L_1 = BD \to 2,3$

$\to L_2 = CD \to 2,3.$

G ⟹ 3 For ⟹ (3L)

$\to L_3 = ABEG \to 1,2,3,4.$

$\to L_4 = ACEG \to 1,2,3,4.$

$\to L_5 = FG \to 1,4.$

(2L) $\to L_6 : H$, 1

$\to L_7 : I$, 4.

2 NTL

~~L_5~~ ✓ $L_1 L_5$ = BDFG → 1,2,3,4

✓ $L_1 L_7$ = BDI → 2,3,4

✓ $L_2 L_5$ = CDFG → 1,2,3,4

✓ $L_2 L_6$ = CDH → 1,2,3.

✓ $L_2 L_7$ = CDI → 2,3,4.

✓ $L_1 L_6$ = BDH → 1,2,3.

~~$L_3 L_7$ = ABEG I → 1,2,3~~

3 NTL

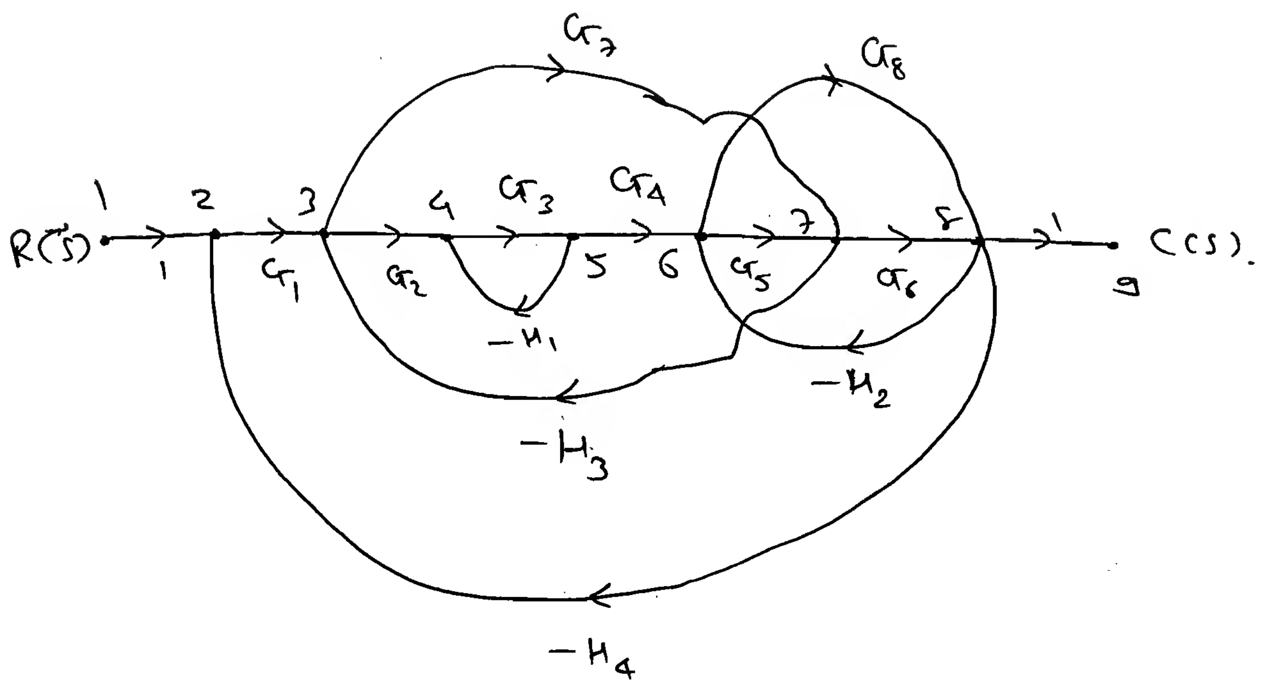
✓ $L_6 L_1 L_7$ = BDIH → 1,2,3,4

✓ $L_6 L_2 L_7$ = CDIH → 1,2,3,4

~~$L_7 L_2 L_6$ = CDHI → 1,2,3,4~~

~~$L_7 L_1 L_6$ = BDHI → 1,2,3,4~~

$\boxed{Q}$



Sol^n:

$H_1 \Rightarrow$ 1 For $4 \leftarrow 5$ $\Rightarrow$ (1L) $\Rightarrow L_1 = -G_3H_1 \rightarrow 4,5.$

$H_2 \Rightarrow$ 2 For $6 \leftarrow 8$ $\Rightarrow$ (2L) $\Rightarrow L_2 = -G_5G_6H_2 \rightarrow 6,7,8.$

$L_3 = -G_8H_2 \rightarrow 6,8.$

$H_3 \Rightarrow$ 2 For $3 \leftarrow 7$ $\Rightarrow$ (2L) $\Rightarrow L_4 = -G_2G_3G_4G_5H_3 \rightarrow 3,4,5,6,7.$

$L_5 = -G_7 \cdot H_3 \rightarrow 3,7.$

$H_4 \Rightarrow$ 3 For $2 \leftarrow 8$ $\Rightarrow$ (3L) $\Rightarrow L_6 = -G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot G_5 \cdot G_6H_4$ $\rightarrow 2,3,4,5,6,7,8.$

$\rightarrow L_7 = -G_1 \cdot G_7 \cdot G_6 \cdot H_4 \rightarrow 2,3,7,8.$

$\rightarrow L_8 = -G_1 \cdot G_2G_3 \cdot G_4 \cdot G_8H_4$ $\rightarrow 2,3,4,5,6,8.$

2NTL

$L_1L_2 \rightarrow 4,5,6,7,8$

$L_1L_3 \rightarrow 4,5,6,7,8$

$L_1L_5 \rightarrow 3,4,5,7$

$L_1L_7 \rightarrow 2,3,4,5,6,7,8.$

$L_3,L_5 \rightarrow 3,6,7,8$

3NTL

$L_1L_3L_5 \rightarrow 3,4,5,6,7,8.$

* Mason's Gain Formula:

Purpose: (i) To find the overall TF of the system.

(ii) To find the ratio of any two nodes.

$$\rightarrow \boxed{\text{overall } TF = \sum_{K=1}^{i} \left(\frac{P_K \cdot \Delta_K}{\Delta} \right).}$$

Where, P_K : K^{th} forward path gain.

$\Delta = 1 - \Sigma$ (individual Loop gain)

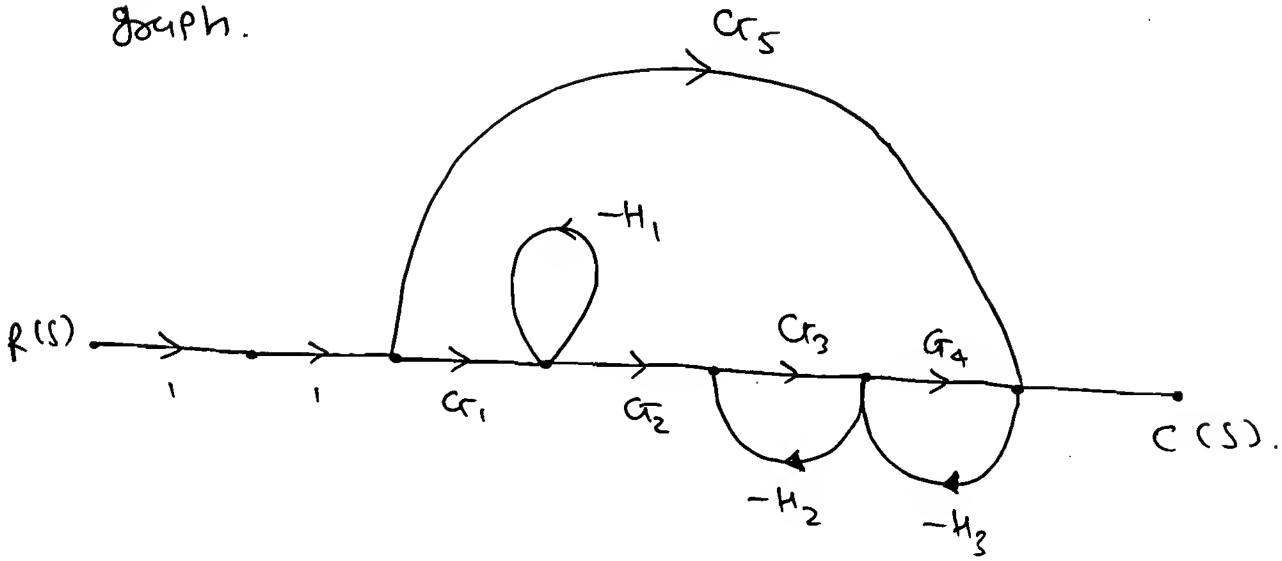
$+ \Sigma$ (Sum of gain product of two non-touching Loop).

$- \Sigma$ (Sum of gain product of three non-touching Loop).

$+ \Sigma$ (sum of gain product of four non-touching Loop). - ....

Δ_K = Δ_K is obtain Δ by removing the Loops touching the K^{th} forward path.

Q. Find the TF to the given signal flow graph.



Sol^n:

F.P.:

$$P_1 = G_1 \cdot G_2 \cdot G_3 \cdot G_4$$

$$P_2 = G_5.$$

Loops:

$$L_1 = -H_1.$$

$$L_2 = -G_3 H_2$$

$$L_3 = -G_4 H_3.$$

2NTL:

$$L_1 L_2 = G_3 H_1 \cdot H_2$$

$$L_1 L_3 = G_4 H_1 \cdot H_3.$$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_1 L_3).$$

$$\therefore \Delta = 1 + H_1 + G_3 H_2 + G_4 H_3 + G_3 H_1 \cdot H_2 + G_4 \cdot H_1 \cdot H_3.$$

$$\Rightarrow \quad \Delta_1 = 1$$

$$\Delta_2 = 1 - (L_1 + L_2) + (L_1 L_2).$$

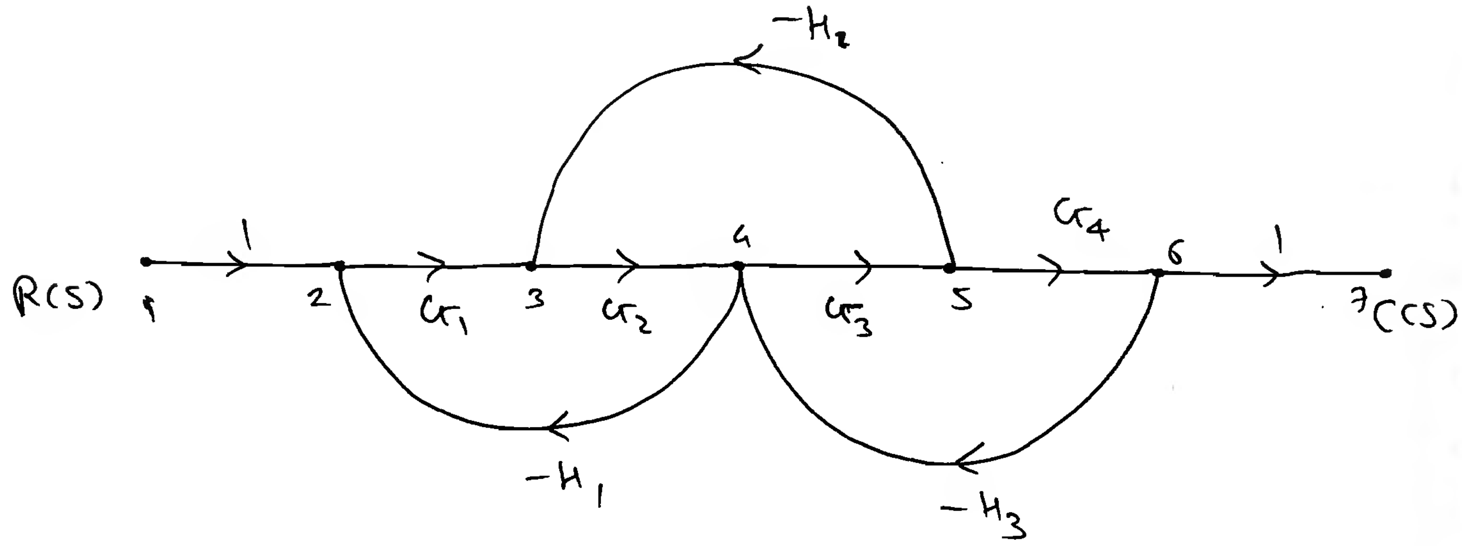
$$\Delta_2 = 1 + H_1 + G_3 H_2 + G_3 H_1 \cdot H_2$$

$$\therefore \quad TF = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_5 (1 + H_1 + G_3 H_2 + G_3 \cdot H_1 H_2)}{1 + H_1 + G_3 H_2 + G_4 H_3 + G_3 H_1 \cdot H_2 + G_4 \cdot H_1 \cdot H_3}.$$

NOTE:

→ In Δ (or) Δ_K, take the opposite sign for odd no. of non-touching loops and take the same sign for even no. of non-touching loops.

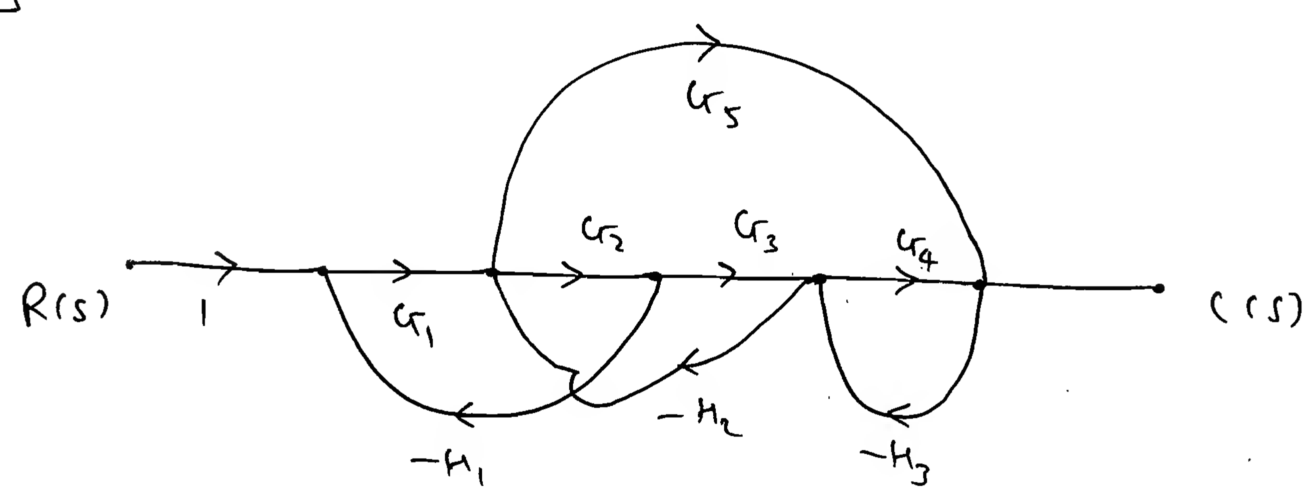
Q-2 Find the TF.



Sol^n:

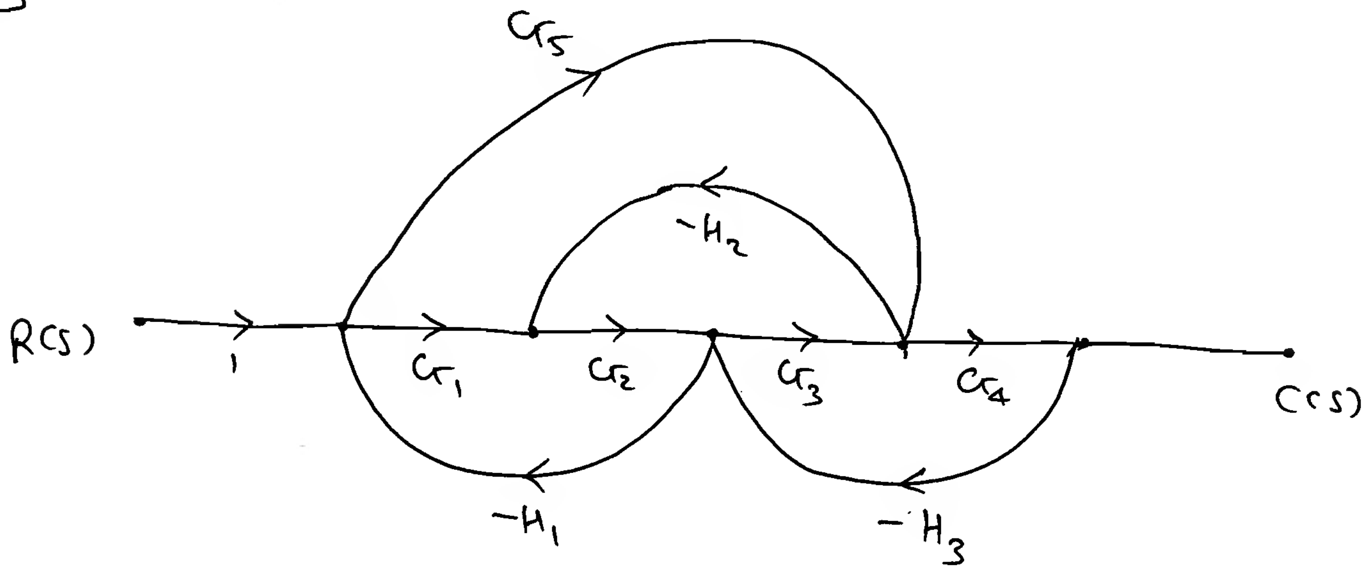
$$TF = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 \, (1)}{1 + G_1 \cdot H_1 + G_3 \cdot H_3 + G_2 \cdot G_3 H_2}$$

Q



$$TF = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_1 \cdot G_5}{1 + G_1 \cdot G_2 H_1 + G_2 \cdot G_3 H_2 + G_4 \cdot H_3 \mp G_5 \cdot H_2 \cdot H_3 + G_1 G_2 \cdot G_4 \cdot H_1 \cdot H_3}$$

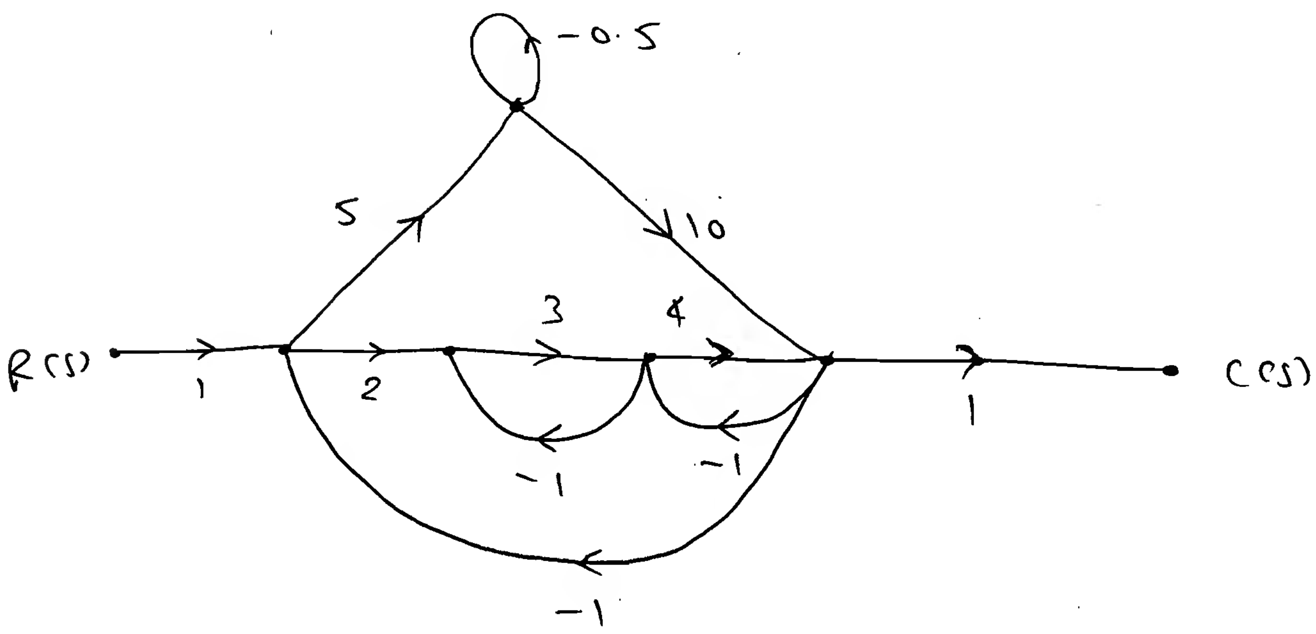
$\boxed{Q}$



Sol^n:

$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_5 G_4}{1 + G_1 G_2 H_1 + G_3 \cdot G_4 \cdot H_3 + G_2 \cdot G_3 H_2 + - G_5 \cdot G_4 \cdot H_3 \cdot H_1 - G_5 \cdot H_2 \cdot G_2 \cdot H_1}$$

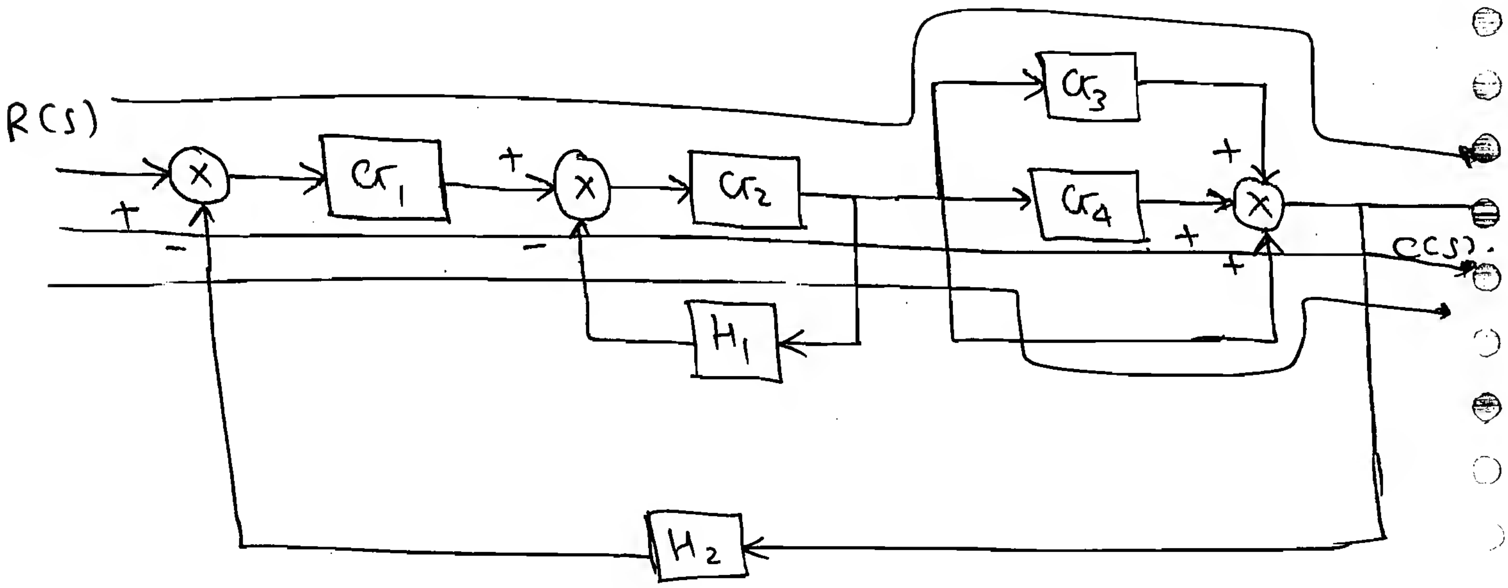
$\boxed{Q}$ Find the TF.



$$\frac{C(s)}{R(s)} = \frac{(2\cdot3\cdot4)(1+0.5) + (5\cdot10)(1+3)}{1+3+4+24+50+0.5+(3/2+2) + (50\cdot3) + (0.5\times24)}$$

$$\frac{C(s)}{R(s)} = \frac{236}{248}.$$

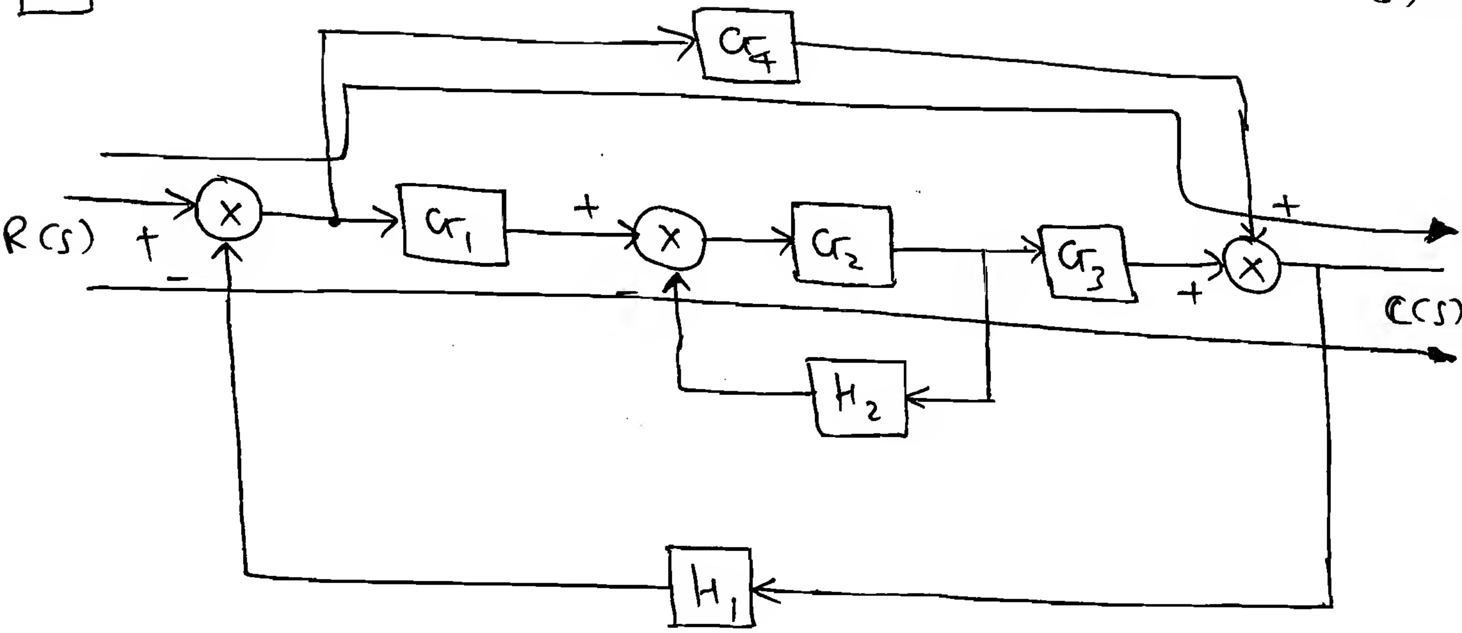
$\boxed{Q}$ Find the TF to the given Block Diagram by using Mason's gain formula.



Sol^n:

$$\frac{C(s)}{R(s)} = \frac{G_1\cdot G_2 G_4 + G_1\cdot G_2\cdot G_3 + G_1\cdot G_2\cdot 1}{1 + G_2H_1 + G_1\cdot G_2\cdot G_3H_2 + G_1\cdot G_2\cdot G_4 H_2 + G_1\cdot G_2\cdot H_2}$$

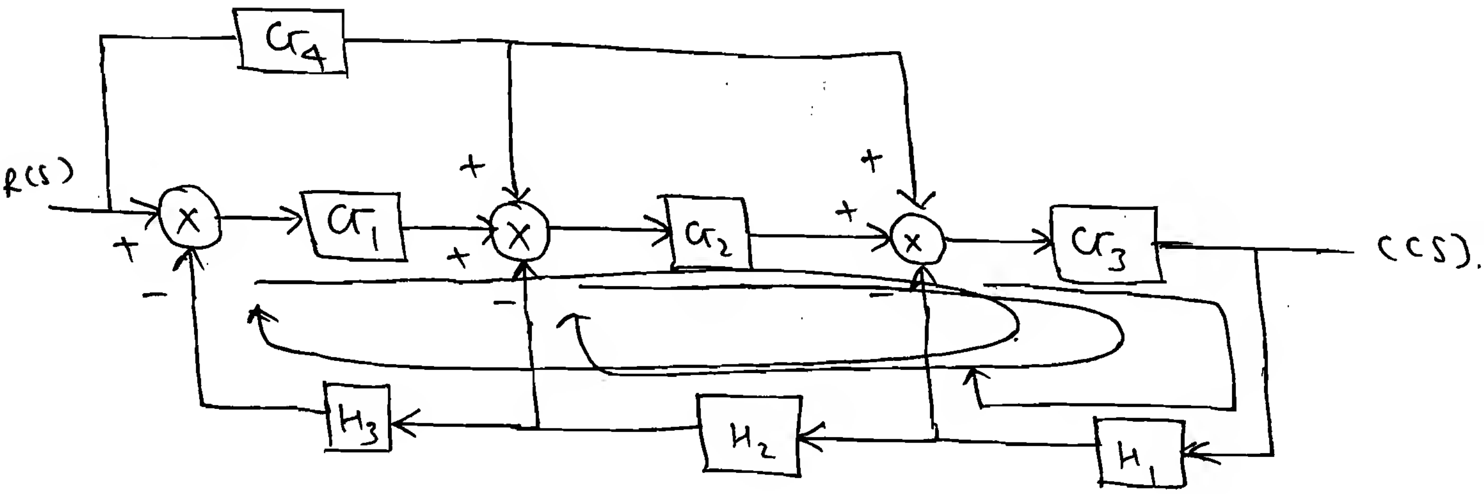
Q



Soln:

$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \cdot G_3 + G_4(1 + G_2 \cdot H_2)}{1 + G_2 \cdot H_2 + G_1 \cdot G_2 \cdot G_3 H_1 + G_4 H_1 + G_4 \cdot H_1 \cdot G_2 \cdot H_2}$$

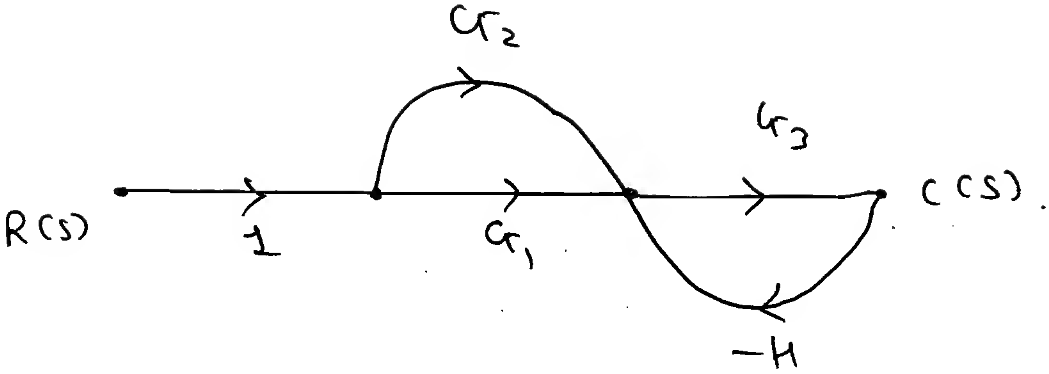
Q



Soln:

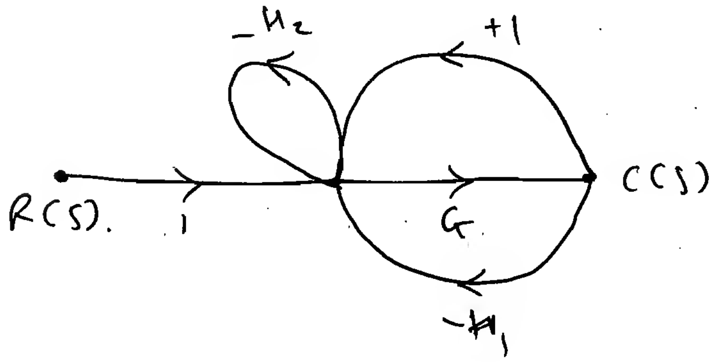
$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \cdot G_3 + G_4 \cdot G_2 \cdot G_3 + G_4 \cdot G_3}{1 + G_3 H_1 + G_2 \cdot G_3 H_1 \cdot H_2 + G_1 \cdot G_2 \cdot G_3 H_1 \cdot H_2 \cdot H_3}$$

$\boxed{Q}$



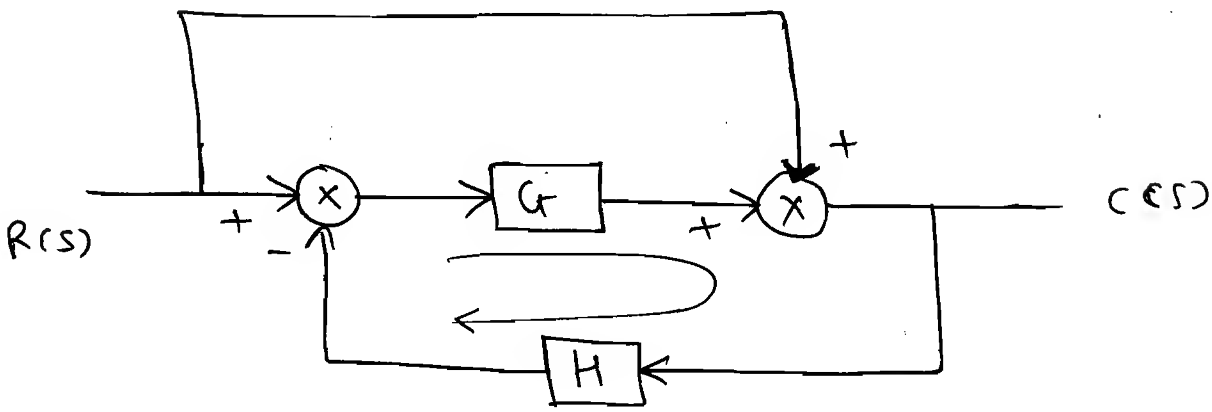
Soln:
$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_3 + G_2 \cdot G_3}{1 + G_3 H}$$

$\boxed{Q}$



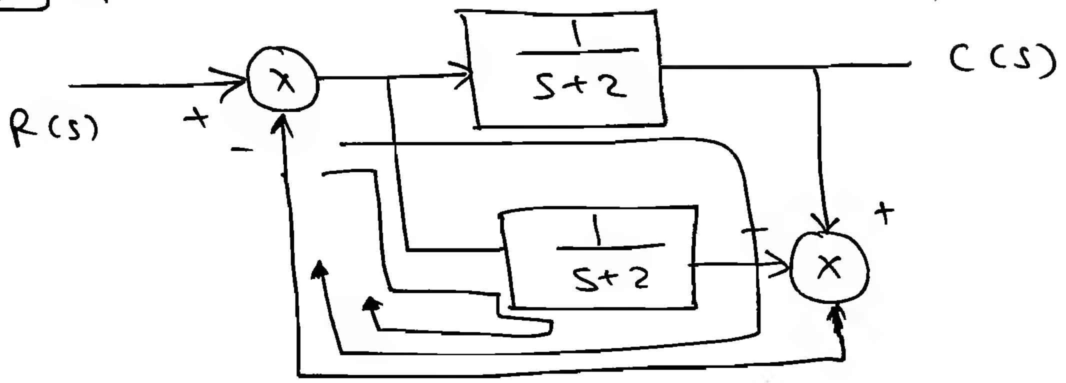
Soln:
$$\frac{C(s)}{R(s)} = \frac{G}{1 + GH_1 + \cancel{G} + H_2}$$

$\boxed{Q}$ *



Soln:
$$\frac{C(s)}{R(s)} = \frac{G+1}{1+GH}$$

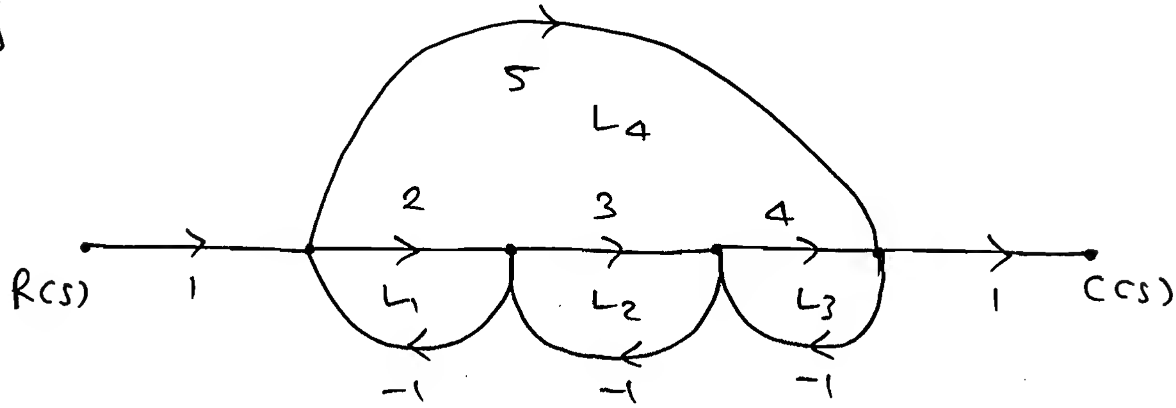
$\boxed{Q}$ *



Soln:

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}-\frac{1}{s+2}} = \frac{1}{s+2}.$$

$\boxed{Q}$

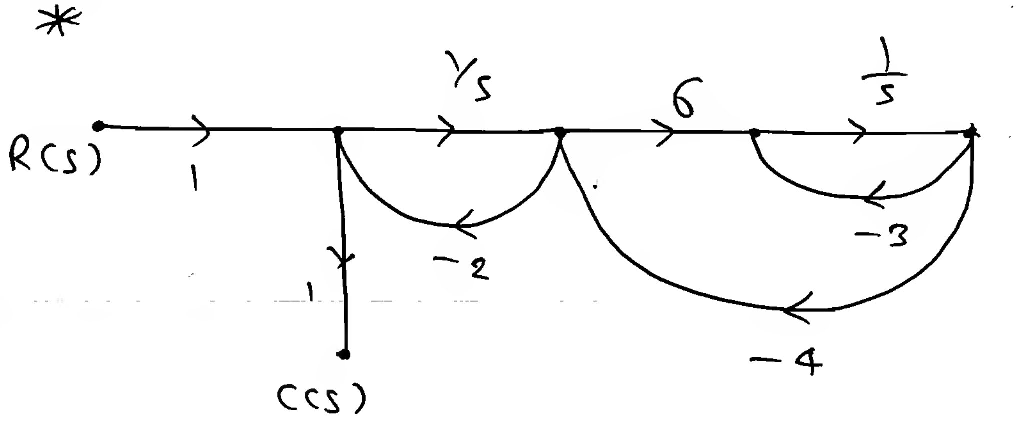


Soln:

$$\frac{C(s)}{R(s)} = \frac{(2\cdot3\cdot4) + (5)(1+3)}{1+2+3+4+8+5}$$

$$\frac{C(s)}{R(s)} = \frac{44}{23}.$$

$\boxed{Q}$ ** *

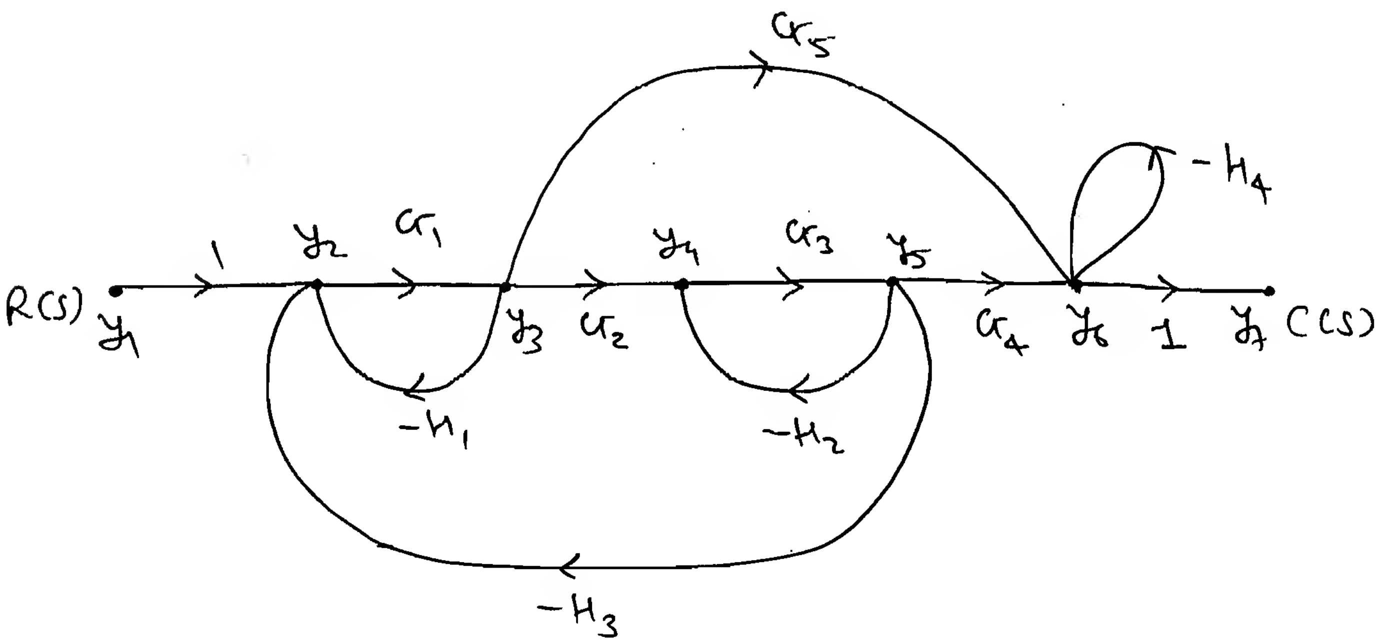


$\underline{\underline{\text{Soln.}}}$ $$\frac{C(s)}{R(s)} = \frac{1.1\left(1 + 3/s + \frac{24}{s}\right)}{1 + \frac{2}{s} + 3/s + \frac{24}{s} + 6/s^2}$$

$$\frac{C(s)}{R(s)} = \frac{s(s+27)}{s^2 + 29s + 6}$$

$\boxed{Q}$ Find $\frac{y_6}{y_1}, \frac{y_7}{y_1}, \frac{y_5}{y_1}, \frac{y_2}{y_1}, \frac{y_7}{y_2}, \frac{y_5}{y_3}, \frac{y_5}{y_4}$

and so on ratio of any two nodes.



$\underline{\underline{\text{Soln:}}}$ (i) $\frac{y_6}{y_1}$

$$\rightarrow \frac{y_6}{y_1} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_1 \cdot G_5(1 + G_3 H_2)}{\left(1 + G_1 \cdot H_1 + G_3 \cdot H_2 + H_4 + G_1 \cdot G_2 \cdot G_3 \cdot H_3 + G_1 \cdot H_1 \cdot G_3 \cdot H_2 + G_1 \cdot H_1 H_4 + G_3 \cdot H_2 \cdot H_4 + G_1 \cdot H_1 \cdot G_3 \cdot H_2 \cdot H_4\right)}$$

Δ

(ii) $\dfrac{y_7}{y_1}$

$\rightarrow$ $y_6 \approx$ $y_7 = 1 \cdot y_6$.

$\therefore \dfrac{y_7}{y_1} = \dfrac{y_6}{y_1}$.

$$\therefore \frac{y_7}{y_1} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_1 \cdot G_5 (1 + G_3 H_2)}{\Delta}$$

(iii) $\dfrac{y_5}{y_1}$

$$\rightarrow \frac{y_5}{y_1} = \frac{G_1 \cdot G_2 \cdot G_3 \, (1 + H_4)}{\Delta}$$

(iv) $\dfrac{y_2}{y_1}$

$$\rightarrow \frac{y_2}{y_1} = \frac{1 \, (1 + G_3 H_2 + H_4 + G_3 H_2 \cdot H_4)}{\Delta}$$

(v) $\dfrac{y_7}{y_2}$

$\rightarrow$ <u>NOTE</u> :$\rightarrow$ The Mason's gain formula gives the ratio w.r.t. input only. It can not gives the ~~nodes~~ ratio directly w.r.t. middle nodes.

$$\rightarrow \frac{y_7}{y_2} = \frac{y_7 | y_1}{y_2 | y_1}$$

$$\rightarrow \frac{y_7}{y_2} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_1 \cdot G_5 (1 + G_3 H_4)}{(1 + G_3 \cdot H_2 + H_4 + G_3 \cdot H_2 \cdot H_4)}$$

$$\rightarrow \frac{y_5}{y_3} = \frac{y_5 | y_1}{y_3 | y_1}$$

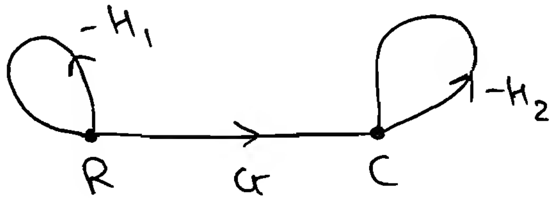
$$\frac{y_5}{y_3} = \frac{G_1 \cdot G_2 \cdot G_3 (1 + H_4)}{G_1 (1 + H_4 + G_3 H_2 + G_3 H_2 H_4)}$$

$$\rightarrow \frac{y_5}{y_4} = \frac{y_5 | y_1}{y_4 | y_1}$$

$$\frac{y_5}{y_4} = \frac{G_1 \cdot G_2 \cdot G_3 (1 + H_4)}{G_1 \cdot G_2 (1 + H_4)}$$

$$* \rightarrow \frac{y_5}{y_4} = G_3.$$

Q

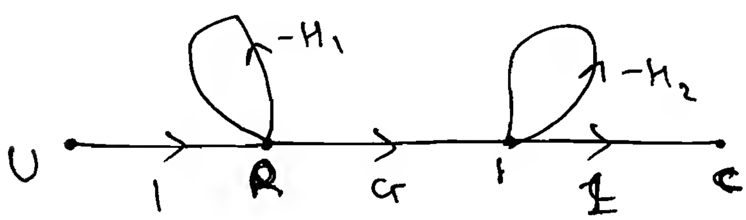


H.B

Soln:

NOTE: In the above signal flow graph R is not input node. In this case we require to assume a dummy input node with path gain of 1 as shown in fig.

$\Rightarrow$



M-I :

$$\frac{C}{R} = \frac{C|U}{R|U} = \frac{G}{1+H_2}.$$

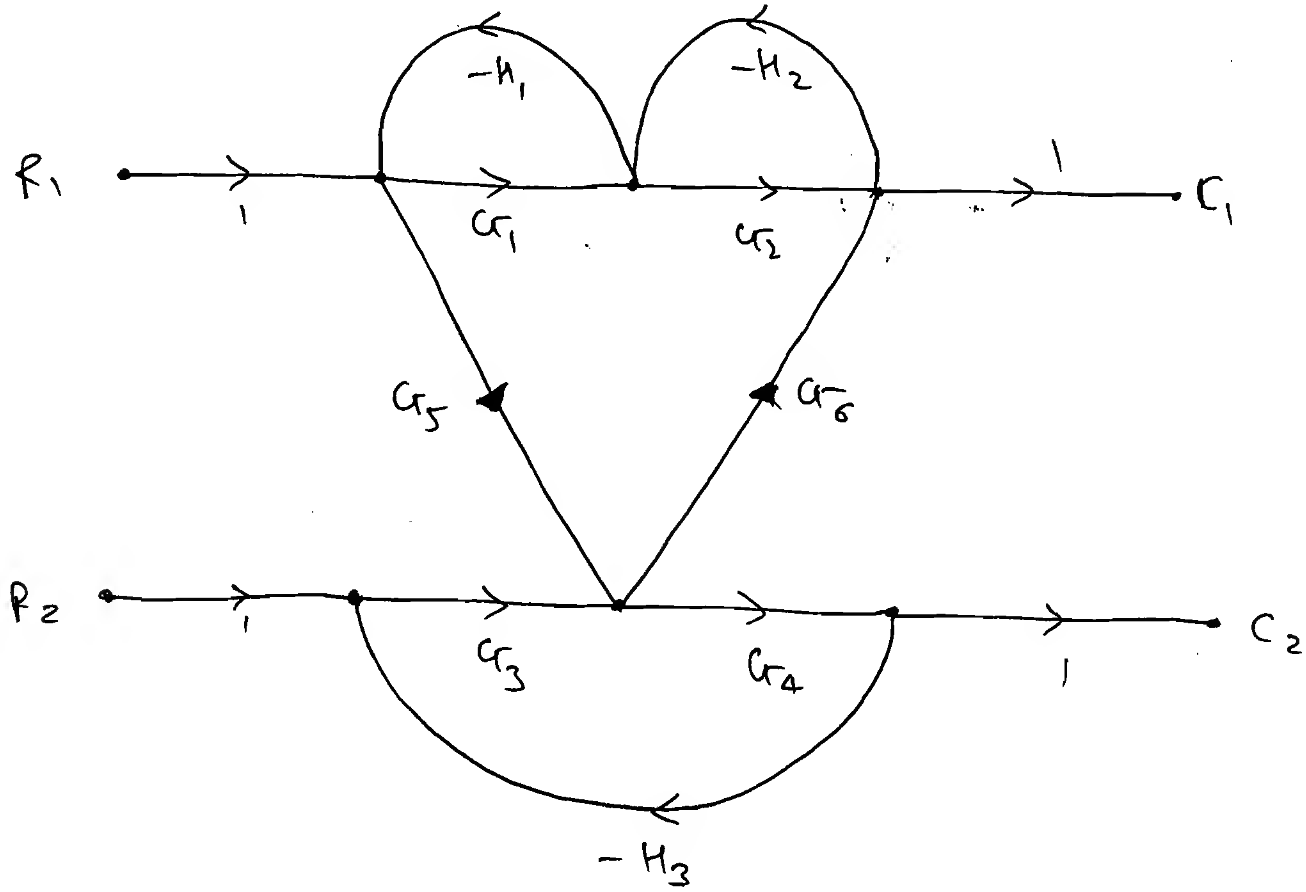
M-II: o/p node eqn.

$$C = RG - CH_2.$$

$$\therefore C(1+H_2) = RG$$

$$\therefore \frac{C}{R} = \frac{G}{1+H_2}.$$

$\boxed{Q}$ Find C_1/R_1, C_1/R_2, C_2/R_1, C_2/R_2 to the given multi-input multi o/p system.



Soln:

$$\frac{C_1}{R_1} = \frac{G_1 \cdot G_2(1 + G_3G_4 \cdot H_3) + G_5 \cdot G_6}{1 + G_1 \cdot H_1 + G_2 \cdot H_2 + G_3G_4 \cdot H_3 - G_5G_6 \cdot H_1 \cdot H_2 + G_1 \cdot H_1 \cdot G_3 \cdot G_4 \cdot H_3 + G_2H_2 \cdot G_3 \cdot G_4 \cdot H_3}$$

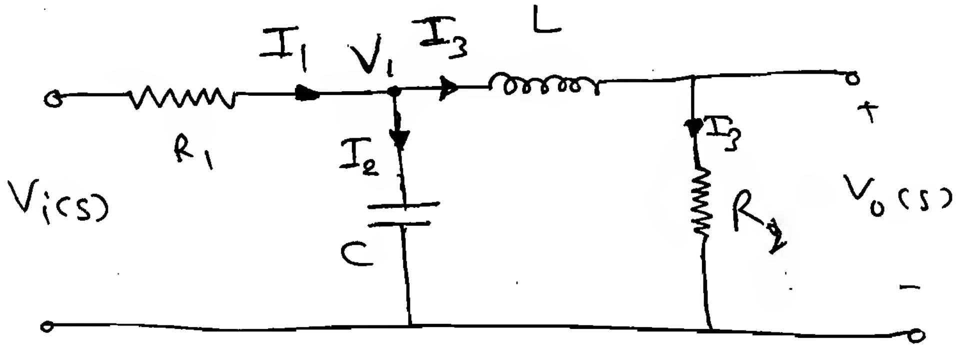
$$\rightarrow \frac{C_1}{R_2} = \frac{G_3 \cdot G_6 \left(1 + G_1 \cdot H_1\right)}{\Delta}.$$

$$\rightarrow \frac{C_2}{R_1} = \frac{G_5 \cdot G_4 \left(1 + G_2 \cdot H_2\right)}{\Delta},$$

$$\rightarrow \frac{C_2}{R_2} = \frac{G_3 \cdot G_4 \left(1 + G_1 H_1 + G_2 H_2\right)}{\Delta}.$$

* Construction of SFG to Electrical N/w:

Q Draw the SFG.



Soln:

→ Select the branch current and node Voltages.

→ Apply Laplace transform to the N/w variable and elements.

→ write the eqns for unknown currents and unknown Voltages.

$$\rightarrow \quad I_1(s) = \frac{V_i(s) - V_1(s)}{R_1}. \quad — (1)$$

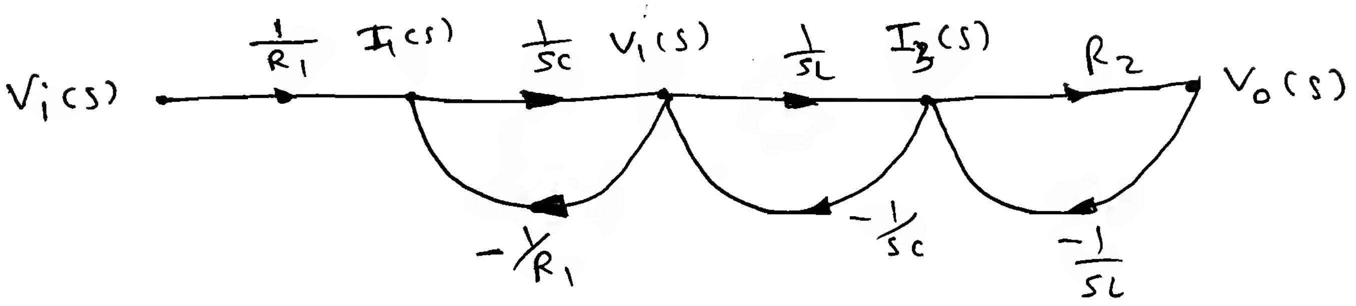
$\therefore \quad V_1(s) = I_2(s) \cdot \frac{1}{sC}$.

But $\quad I_2(s) = I_1(s) - I_3(s)$.

$\rightarrow \therefore \quad V_1(s) = \frac{1}{sC}\left(I_1(s) - I_3(s)\right) \quad — ②$

$\rightarrow \quad I_3(s) = \frac{V_1(s) - V_0(s)}{sL}. \quad — ③$

$\rightarrow \quad V_0(s) = R_2 \cdot I_3(s). \quad — ④$



$\Rightarrow \quad \boxed{TF_{E\text{-}N/W} = TF_{B\text{-}D} \text{ (or) } TF_{SFG}}$

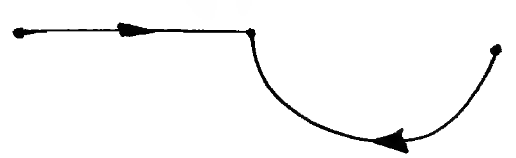
* **Procedure to draw SFG directly.**

→ The nodes in a SFG are nothing but the variable along the series path. (branch).

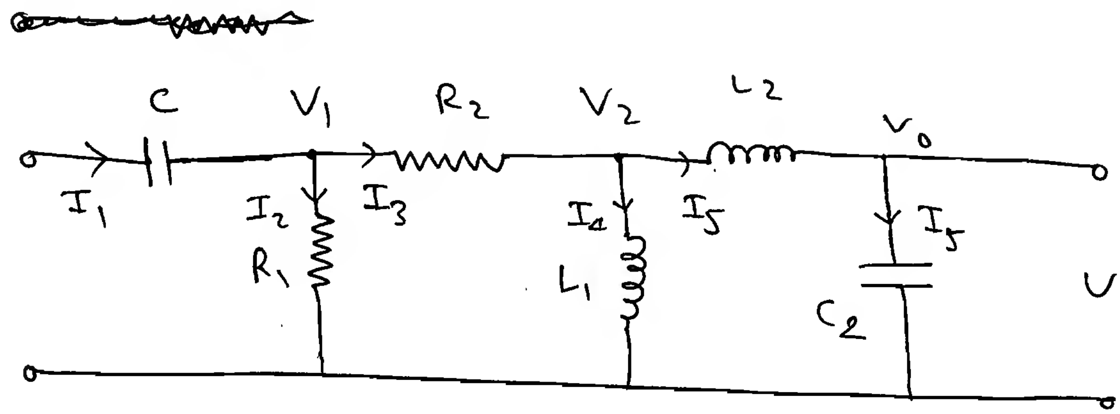
→ Each Elements in electrical N/W gives the 1 Forward path and 1 -ve feedback path. except the last element. The last element is giving the only

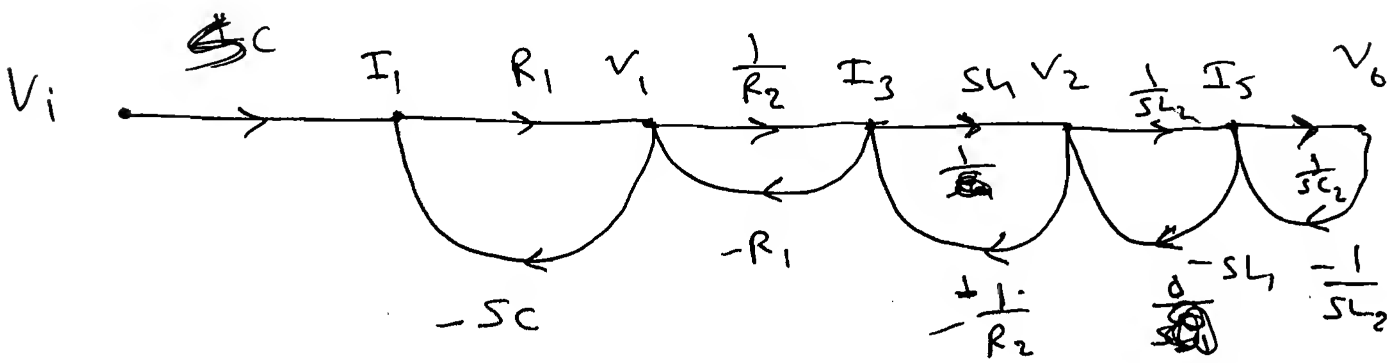
forward path.

→ Take inverse of the impedence to the series branch elements as a path gain and take the same impedence for shunt branch elements as a path gain.

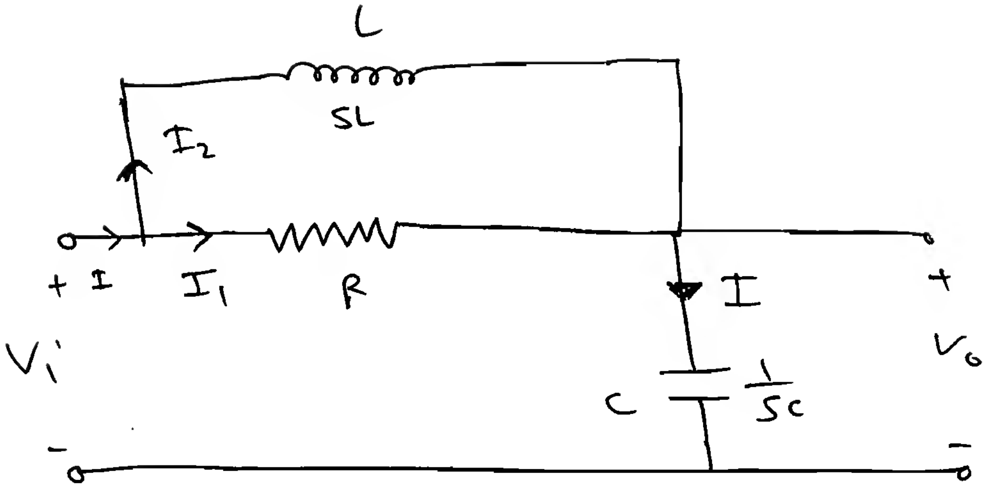
⇒ 

$\boxed{a}$





Q
* * *

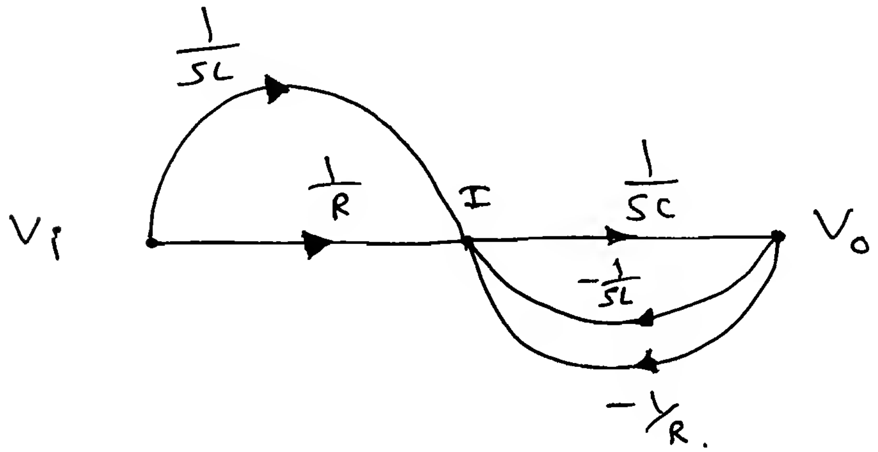


Soln:

$$V_o = \frac{1}{sC} \cdot I.$$

$$I = I_1 + I_2$$

$$I = \frac{V_i - V_o}{R} + \frac{V_i - V_o}{sL}.$$



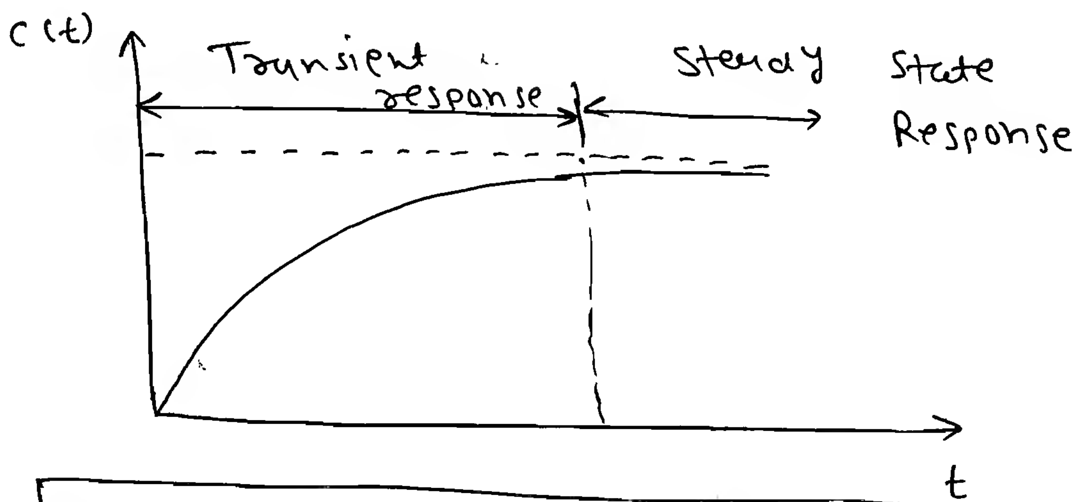
☆ Time Domain Analysis:

→ **Purpose:** To evaluate the performance of the system w.r.t. to the time.

* Time - Response:-

→ If the response of the system varies with respect to the time then it is called as time response.

→ The time response is nothing but the sum of transient response and steady state response.

⇒ 

$$\boxed{TR\,(c(t)) = C_{tr}(t) + C_{ss}(t)}$$

⇒ Find the transient and steady state terms in the given time response.

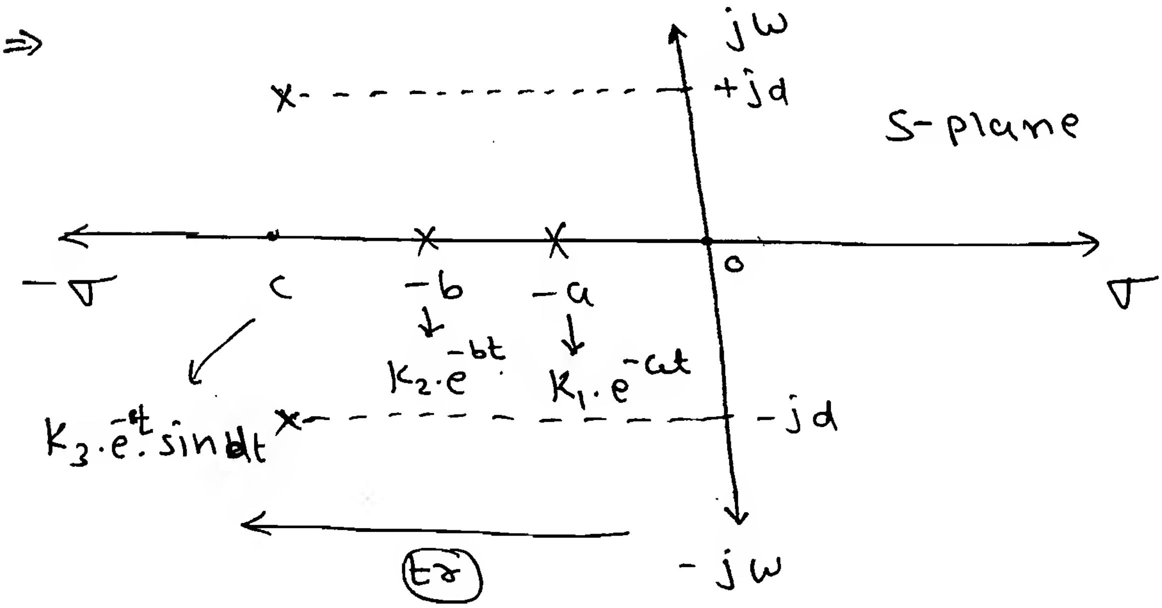
$$C(t) = \underbrace{10 + 2\sin 2t + 3\cos 3t}_{S.S} + \underbrace{4t\cdot e^{-4t} + 5\cdot e^{-5t}\cdot \sin 5t + 6t\cdot e^{-6t}\cdot \cos 6t}_{tr}$$

⇒ **Transient term:**

⇒ It is part of the system that becomes 0 as t becomes very large.

i.e. $$\boxed{\lim_{t \to \infty} C_{tr}(t) = 0}$$

⇒



⇒ The term which consist exponentially decay always gives the transient terms.

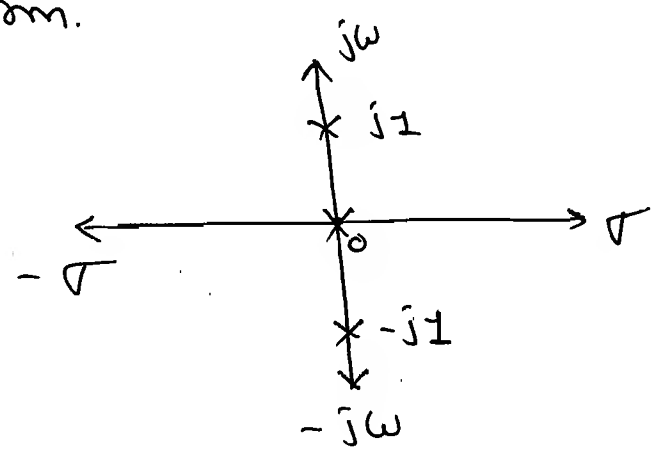
⇒ The poles which lies left hand side of the s-plane gives the transient terms.

⇒ Steady State Response:

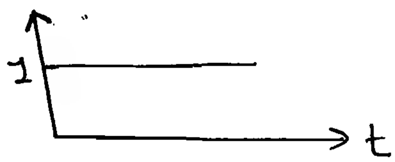
⇒ It is the part of the response that remains after the transient becomes the zero.

→ The pole which lies on the imaginary axis gives the steady state term.

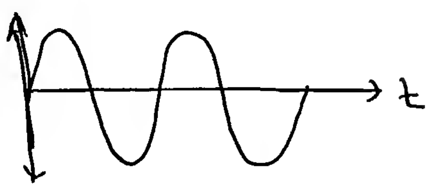
⇒



→ $\frac{1}{s} \Rightarrow u(t)$

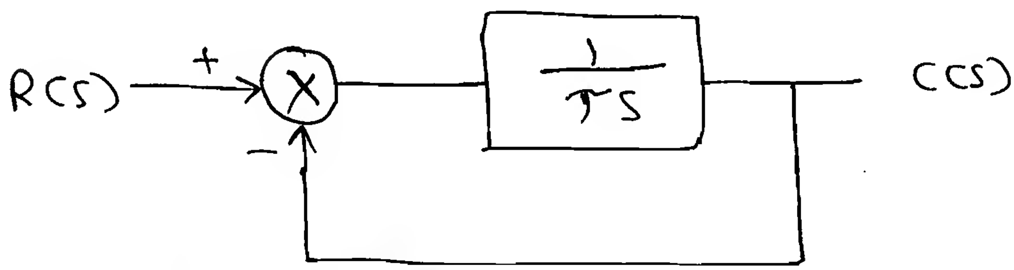


→ $\frac{1}{s^2+1} \rightarrow \sin t$



* Time Response to the first order system.

⇒



→ $G(s) = \frac{1}{\tau s}$, $H(s) = 1$.

∴ Type - 1 & order - 1.

⇒ Practical ckt for the first order system is RC, RL ckt of LPF.

→ <u>Impulse response:</u>

→ $r(t) = \delta(t)$.

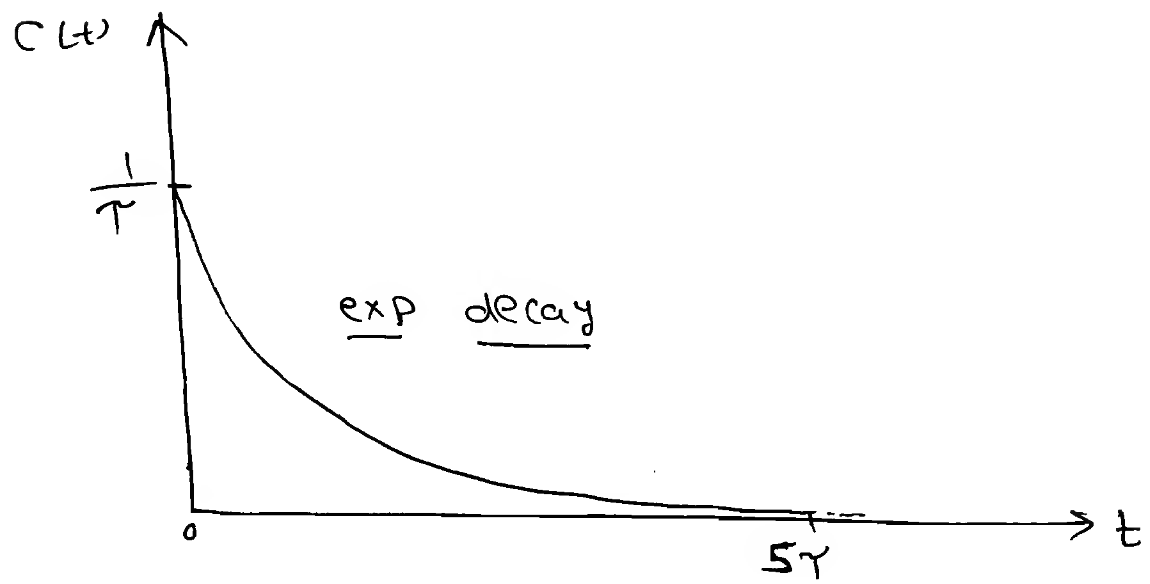
⇒ $R(s) = 1$.

→ $\frac{C(s)}{R(s)} = \frac{1}{\tau s+1}$.

→ $C(s) = \frac{1}{\tau s+1}$.

$= \frac{1}{\tau} \cdot \frac{1}{s+\frac{1}{\tau}}$.

→ $\boxed{c(t) = \frac{1}{\tau} \cdot e^{-t/\tau}}$ → Transient term.

⇒ 

→ The impulse response consist the transient term. Transient term consist the

the System parameters.

→ Hence, the impulse response is called system response (or) Natural response (or) ~~force~~ free forced response.

* error:

→ error is nothing but the deviation of the output from the input.

i.e. $e(t) = r(t) - c(t)$.

* Steady State error (e_{ss}):

→ The error at $t \to \infty$.

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t).}$$

→ $e_{ss} = \lim_{t \to \infty} r(t) - c(t).$

$$= \lim_{t \to \infty} \delta(t) - \frac{1}{T} \cdot e^{-t/T}.$$

→ The impulse ~~response~~ not consist the any steady state terms. Hence we can not defined the steady state errors.

(or) The impulse input not exist at $t \to \infty$.

Hence we can not compare the o/p with

i/p at $t \to \infty$.

⇒ <u>Unit</u> <u>Response</u>:

⇒ $r(t) = u(t) \Rightarrow R(s) = \frac{1}{s}$.

$$\frac{C(s)}{R(s)} = \frac{1}{(\tau s+1)}.$$

$$\therefore C(s) = \frac{1}{s(1+s\tau)}.$$

$$1 = \frac{A}{s} + \frac{B}{(1+s\tau)}.$$

$$\therefore 1 = A(1+s\tau) + Bs.$$

$s \to 0$. $A = 1$

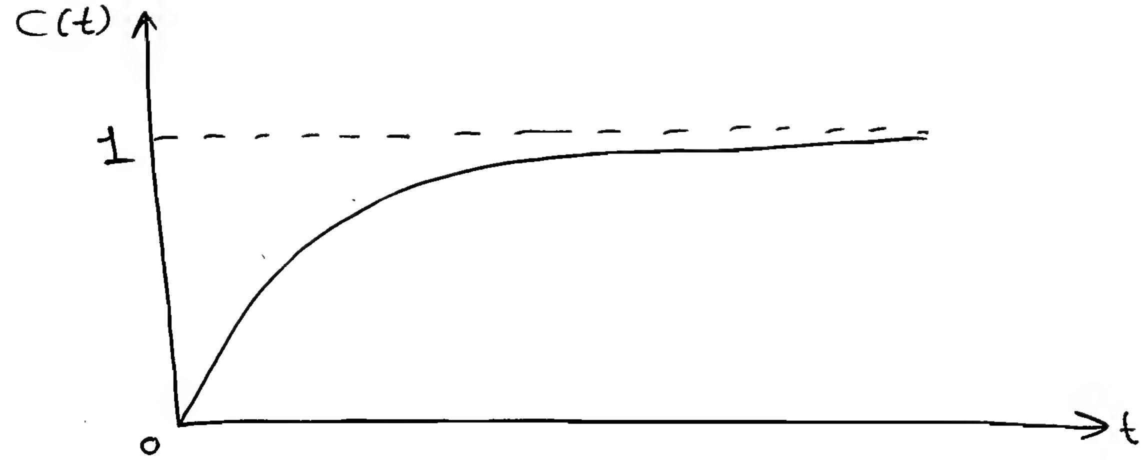
$s \to -1/\tau$ $B = -\tau$.

$$\therefore C(s) = \frac{1}{s} - \frac{\tau}{1+s\tau} = \frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}.$$

$$\therefore \boxed{c(t) = \left(1 - e^{-t/\tau}\right)u(t).}$$

SS tr

→ In the response, the steady state term because of the input in the response and the transient term because of the system.



$\Rightarrow \quad e_{ss} = \lim_{t\to\infty} r(t) - c(t)$

$= \lim_{t\to\infty} 1 - 1 + e^{-t/\tau}$

$\therefore \boxed{e_{ss} = 0}$

$\Rightarrow$ Unit Ramp Response:

$\Rightarrow \quad r(t) = t \quad \Rightarrow \quad R(s) = \frac{1}{s^2}.$

$\therefore \quad C(s) = \frac{1}{s^2} \cdot \frac{1}{(s\tau+1)}.$

$\rightarrow \quad \frac{1}{s^2(s\tau+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s\tau+1)}.$

$\rightarrow \quad 1 = As(s\tau+1) + B(s\tau+1) + Cs^2.$

$\rightarrow s \to 0$, $\rightarrow s \to -\frac{1}{\tau}$

$\therefore \boxed{1 = B}$ $\quad 1 = \frac{C}{\tau^2} \Rightarrow \boxed{C = \tau^2}.$

$s \to 1$

$\therefore 1 = A(\tau+1) + B(\tau+1) + C.$

$\Rightarrow \cancel{1} = A(\tau+1) + \tau + \cancel{1} + \tau^2.$

$\therefore \quad A\,(\tau \cancel{+1}) = -\tau\,(1 \cancel{+\tau})$

$\boxed{A = -\tau}$

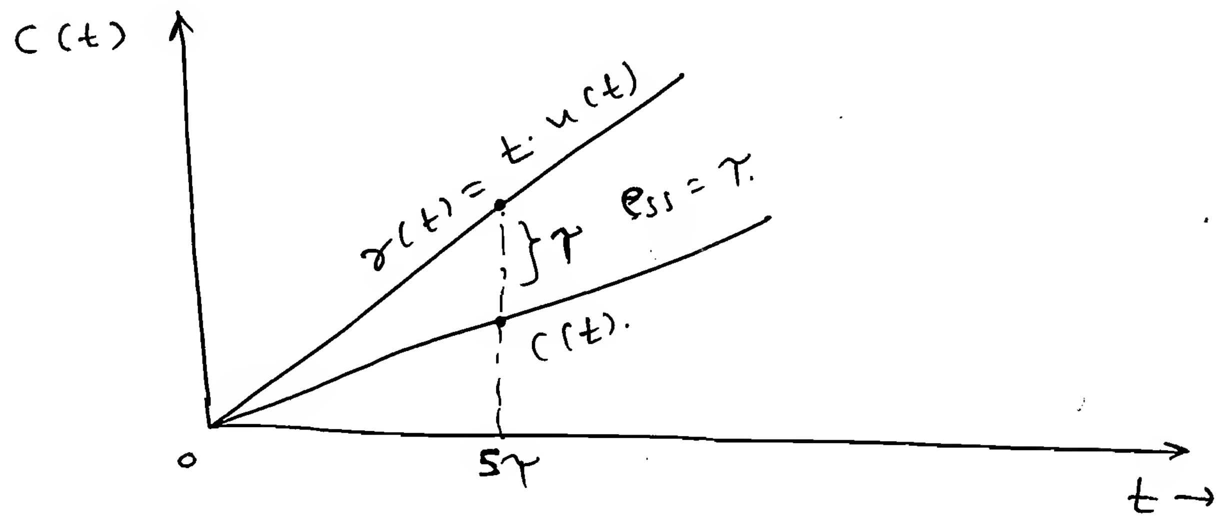
$$\therefore \quad C(s) = \frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{\tau s+1}.$$

$$\therefore \quad \boxed{c(t) = t - \tau + \tau \cdot e^{-t/\tau}}.$$

$$\rightarrow e_{ss} = \lim_{t\to\infty} r(t) - c(t).$$

$$= \lim_{t\to\infty} \tau - e^{-t/\tau} = \tau.$$

$$\therefore \quad \boxed{e_{ss} = \tau}$$



$\Rightarrow$ <u>Unit</u> <u>Parabolic</u> <u>Response:</u>

$\Rightarrow \quad r(t) = t^2/2 \cdot u(t).$

$\Rightarrow \quad R(s) = \dfrac{1}{s^3}.$

$\rightarrow \quad C(s) = \dfrac{1}{s^3} \cdot \dfrac{1}{(s\tau+1)}.$

$$\therefore \frac{1}{s^3(s\tau+1)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s^1} + \frac{D}{s\tau+1}.$$

$$\therefore 1 = A(s\tau+1) + Bs(s\tau+1) + Cs^2(s\tau+1) + Ds^3.$$

$\rightarrow s \rightarrow 0$

$\Rightarrow \boxed{1 = A}$

$\rightarrow s \rightarrow -\frac{1}{\tau}.$

$1 = -\frac{D}{\tau^3} \Rightarrow \boxed{D = -\tau^3}$

~~8t~~

$\rightarrow$ Co-efficient of s.

$A\tau + B = 0. \Rightarrow \cancel{\boxed{B = -\frac{1}{\tau}}} \quad \boxed{B = -\tau}$

$\rightarrow$ Co-efficient of s^2.

$B\tau + C\tau = 0.$

$C = -B\tau$

$\boxed{C = \tau^2}$

$$\therefore C(s) = \frac{1}{s^3} - \frac{\tau}{s^2} + \frac{\tau^2}{s} - \frac{\tau^3}{s\tau+1}.$$

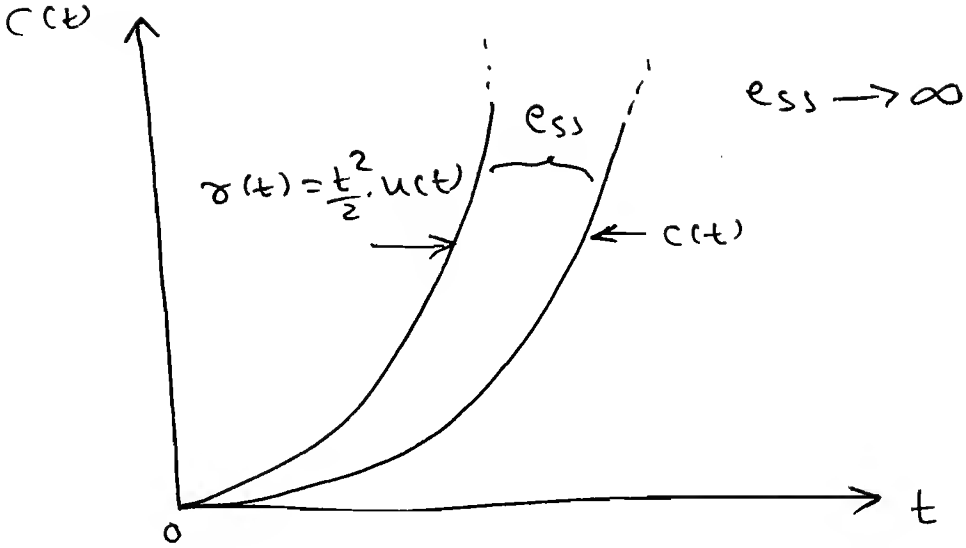
$$\therefore \boxed{c(t) = \frac{t^2}{2} - t\tau + \tau^2 - \tau^2 . e^{-t/\tau}}$$

$$e_{ss} = \lim_{t\to\infty} r(t) - c(t)$$

$$= \lim_{t\to\infty} t\tau - \tau^2 + e^{-t/\tau}.$$

$\boxed{e_{ss} = \infty.}$

$\Rightarrow$

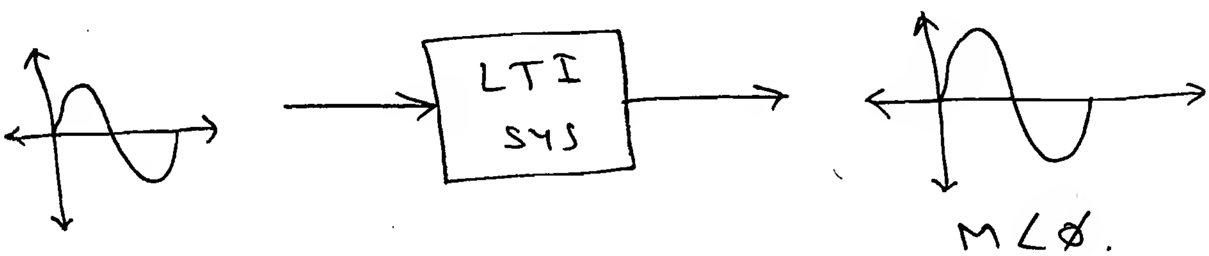


* Sinusoidal Response:

$\Rightarrow$ For any LTI system if input is sinusoidal the o/p also sinusoidal but difference in magnitude and phase.

$\Rightarrow$ The standard form of i/p and o/ps are as follows:

$\Rightarrow$



$\Rightarrow$ $r(t) = A \sin(\omega t \pm \theta) \rightarrow c(t) = A \times M \sin(\omega t \pm \theta \pm \phi)$.

$r(t) = A \cos(\omega t \pm \theta) \longrightarrow c(t) = A \times M \cos(\omega t \pm \theta \pm \phi)$.

$\boxed{Q}$ The CLTF of a LTI system $\frac{C(s)}{R(s)} = \left(\frac{1}{s+1}\right)$. for i/p $r(t) = \sin(t)$ the steady state o/p is ?

Sol^n: $r(t) = \sin(t)$.

$\Rightarrow \quad \omega = 1$ rad/sec.

$$\frac{C(s)}{R(s)} = H(s) = \frac{1}{s+1}.$$

$$\Rightarrow H(j\omega) = \frac{1}{1+j\omega} = \frac{1}{1+j} = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$\therefore \boxed{c(t) = \frac{1}{\sqrt{2}} \cdot \sin(t - 45^\circ).}$$

$\boxed{a}$ $\dfrac{C(s)}{R(s)} = \dfrac{s+1}{s+2}$, $\quad r(t) = 10\cos(2t + 45^\circ)$.

find $c(t) = ?$

Sol^n: $r(t) = 10\cos(2t + 45^\circ)$

$\Rightarrow \quad \omega = 2$ rad/sec.

$$\therefore H(s) = \frac{s+1}{s+2} = \frac{1+j\omega}{2+j\omega} = \frac{1+2j}{2+2j}$$

$$\therefore H(j\omega) = \frac{\sqrt{5}\ \tan^{-1}(2)}{\sqrt{8}\ \tan^{-1}(1)} = \sqrt{\frac{5}{8}} \angle\left(\tan^{-1}(2) - 45^\circ\right)$$

$$\therefore H(j\omega) = \sqrt{\frac{5}{8}} \angle 18.43^\circ.$$

$$\therefore c(t) = 10 \times \sqrt{5/8} \cdot (2t + 45^\circ + 18.43^\circ).$$

$$\therefore \boxed{c(t) = 8\cos(2t + 63.43^\circ).}$$

$\boxed{Q}$ A System $\frac{Y(s)}{X(s)} = \frac{s}{(s+p)}$ as an o/p 10.9

$y(t) = 1.\cos(2t - \pi/3)$. when

$x(t) = p.\cos(2t - \pi/2)$ then the

System parameter p is.

Soln:

$\omega = 2$ rad/sec.

$$\therefore H(s) = \frac{j\omega}{j\omega + p} = \frac{j2}{p+2j} = \frac{2\angle 90^\circ}{\sqrt{p^2+4}\angle\tan^{-1}\left(\frac{2}{p}\right)}$$

$$\therefore H(j\omega) = \frac{2}{\sqrt{p^2+4}} \times \angle 90^\circ - \tan^{-1}\left(\frac{2}{p}\right).$$

Now, $y(t) = x(t).h(t)$.

$$\therefore 1.\cos\left(2t - \frac{\pi}{3}\right) = \frac{2.p}{\sqrt{p^2+4}}.\cos\left(90^\circ + 2t - \frac{\pi}{2} - \tan^{-1}\left(\frac{2}{p}\right)\right)$$

compare, mag. and phase,

$$\frac{2.p}{\sqrt{p^2+4}} = 1$$

$$\therefore 2p = \sqrt{4+p^2}$$

$$4p^2 = 4+p^2$$

$$3p^2 = 4$$

$$\therefore p^2 = \frac{4}{3}$$

$$\boxed{p = \pm 2/\sqrt{3}}$$

$$\therefore -\frac{\pi}{3} = -\tan^{-1}\left(\frac{2}{p}\right)$$

$$\therefore \tan^{-1}\left(\frac{2}{p}\right) = \pi/3.$$

$$\frac{2}{p} = \tan(\pi/3).$$

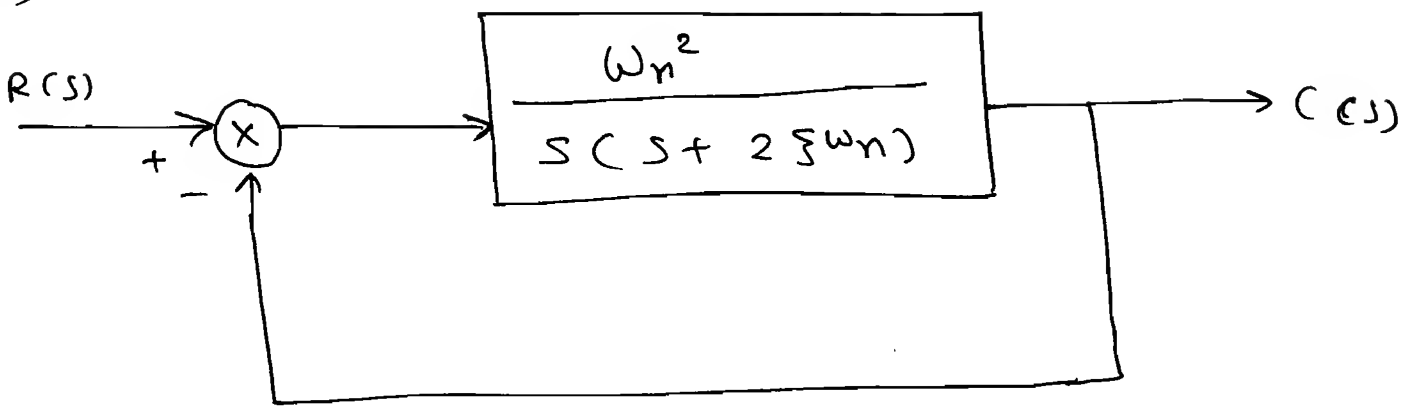
$$\therefore \frac{2}{p} = \sqrt{3}$$

$$\Rightarrow \boxed{p = \cancel{\sqrt{3}/2}}$$

$$p = 2/\sqrt{3}.$$

* Time Response to the second order system :

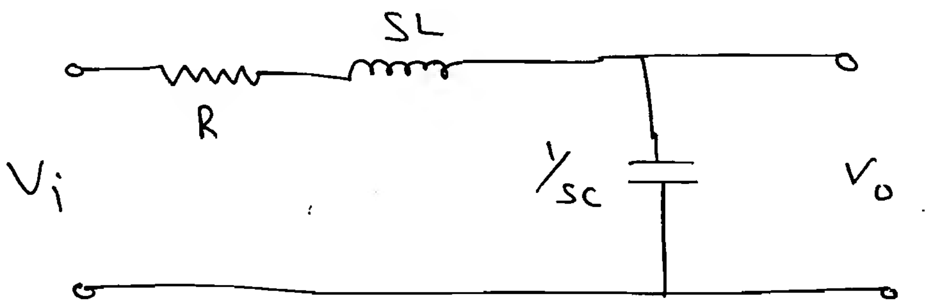
⇒



⇒ $$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}}$$ ← (H.B.)

⇒ Type - 1, order - 2.

⇒ The Practical ckt to the second order system is R-L-C ckt of LPF.



⇒ $$\frac{V_o(S)}{V_i(S)} = \frac{\frac{1}{SC}}{R + SL + \frac{1}{SC}}.$$

$$\frac{V_o(S)}{V_i(S)} = \frac{1}{S^2LC + SCR + 1}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}.$$

$$\Rightarrow \omega_n^2 = \sqrt{\frac{1}{LC}}$$

$$\Rightarrow \omega_n = \frac{1}{\sqrt{LC}}$$

$$2\xi\omega_n = \frac{R}{L}.$$

$$\therefore 2\xi = \frac{R}{L} \times \sqrt{LC}$$

$$\therefore \boxed{\xi = \frac{R}{2} \times \sqrt{\frac{C}{L}}.}$$

$\Rightarrow$ ω_n: is called Natural freq. of oscillation (or) Sustained oscillation (or) Undamped oscillation.

$$\Rightarrow \boxed{\alpha = \frac{1}{2\xi} = \frac{1}{R} \times \sqrt{\frac{L}{C}} = \text{damping ratio.}}$$

$\Rightarrow$ It gives the ratio of energy loosed to energy stored.

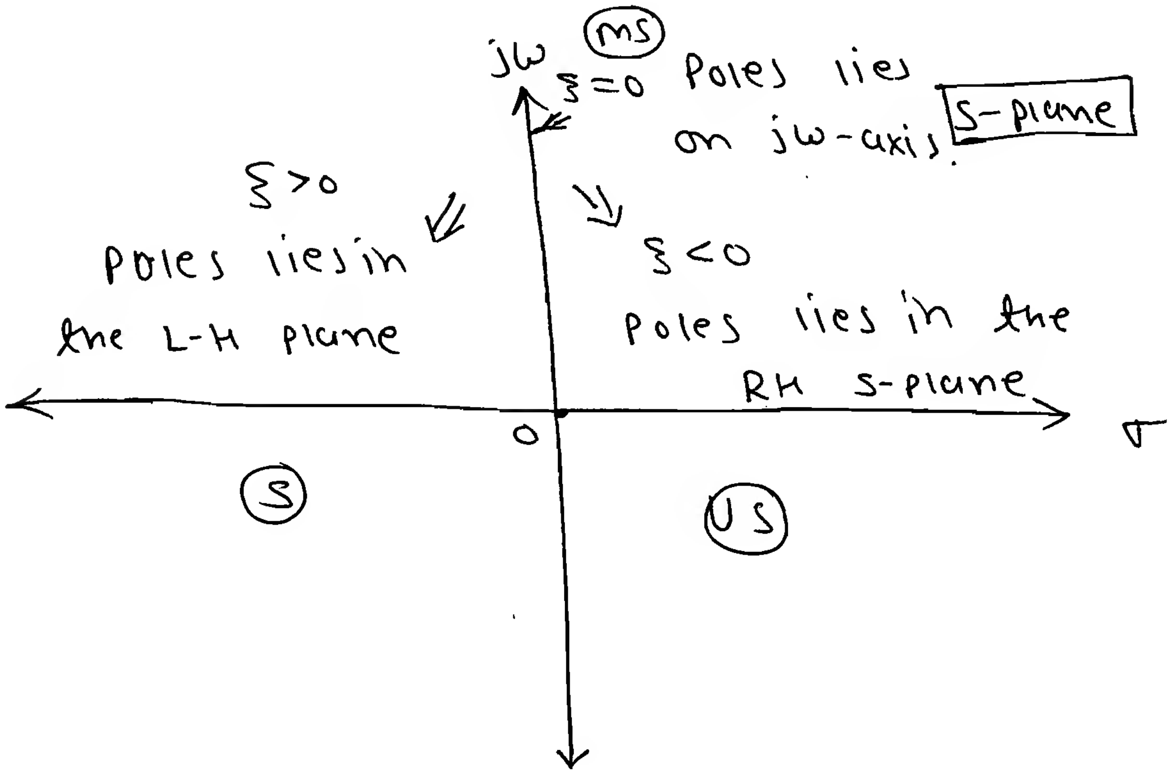
$$\Rightarrow \boxed{\xi\omega_n = \text{damping factor (or) actual damping.}}$$

2) The second order stable for $\xi > 0$.

$\Rightarrow$ The Second order system response completly depends on ξ.

⇒ The Second order system is stable for all the +ve values of $\xi > 0$. because the poles lies in the LH-plane.

⇒



* Impulse Response:

⇒ $r(t) = \delta(t)$.

⇒ $R(s) = 1$.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}.$$

$$\therefore C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}.$$

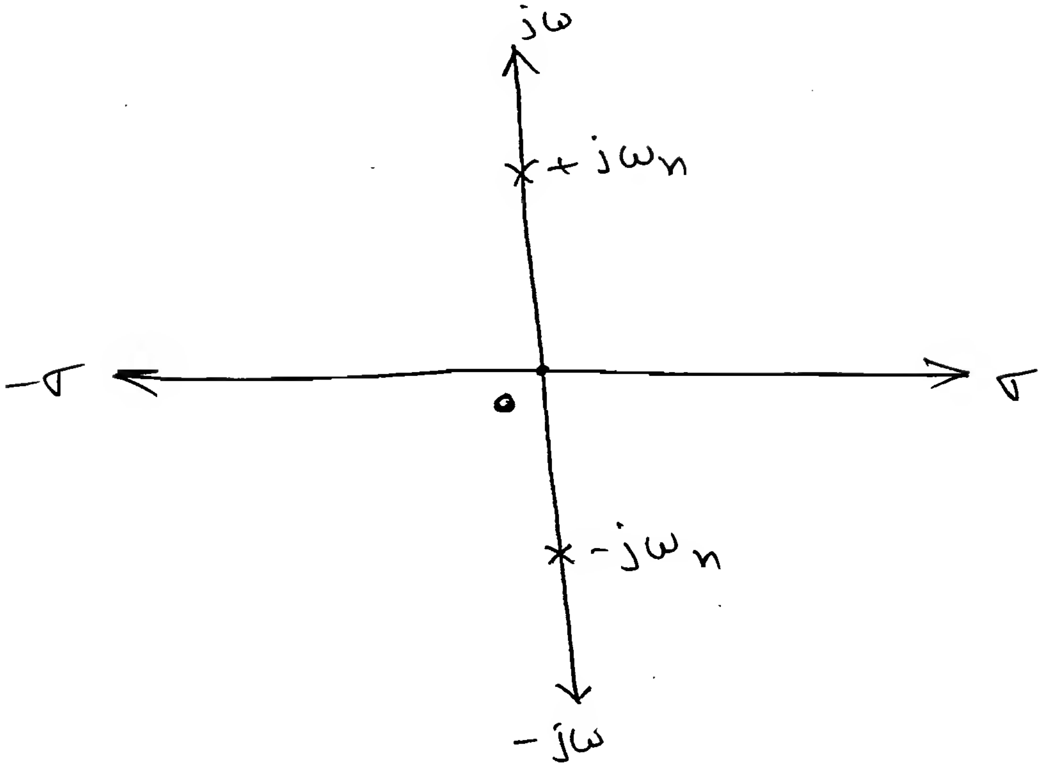
Case - I: $\xi = 0$ ⇒ Undamped.

$$\therefore C(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}.$$

$\therefore$ Poles: $s^2 + \omega_n^2 = 0$

$$\boxed{s = \pm j\omega_n}$$

$\Rightarrow$

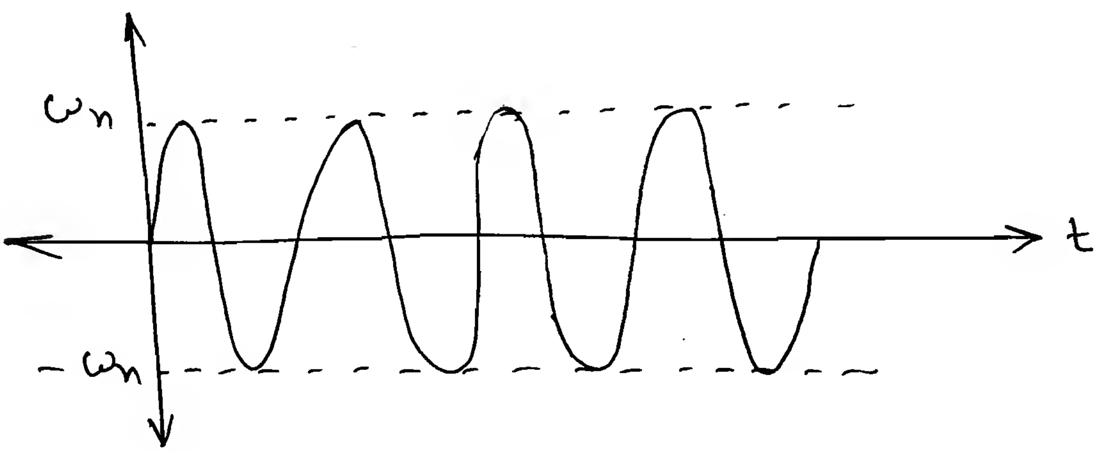


$\Rightarrow$ By Real part:

$\tau = 1/0 = \infty.$

$\Rightarrow$ Non-repeated poles on the $j\omega$ axis hence the system is Marginally stable.

By ILT: $c(t) = \omega_n \cdot \sin\omega_n t.$



Constant Amp. and freq. of oscillation.

undamped oscⁿ | Natural oscⁿ | sustained oscⁿ.

→ When $\xi = 0$, the second order system response is constant amplitude and freq. of oscillation which are called undamped oscillation.

→ Any system which produced the undamped oscillation is called undamped system and the system becomes marginal stable.

→ The second order system nature completely depends on ξ. for example if $\xi = 0$ the second order system nature is constant amplitude and freq. of oscillⁿ around the input which never be changed by changing the input signal hence when $\xi = 0$ the second order system is called undamped system irrespective of all the inputs.

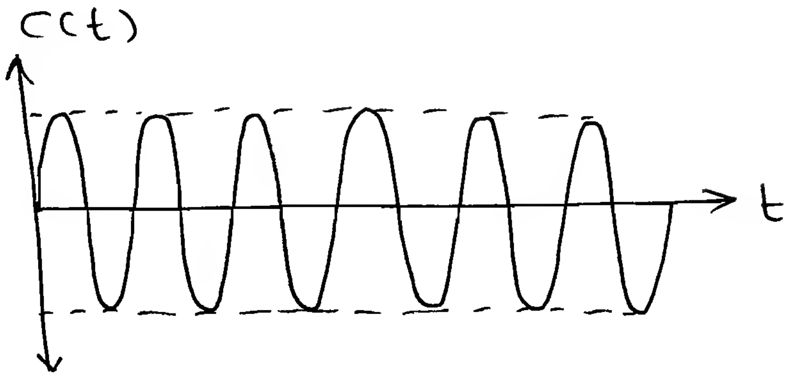
⇒ Similarly when $\xi > 0$ & $\xi < 1$ the system is called under damped system.

$\Rightarrow$ $\xi = 1 \rightarrow$ Critical damped system.

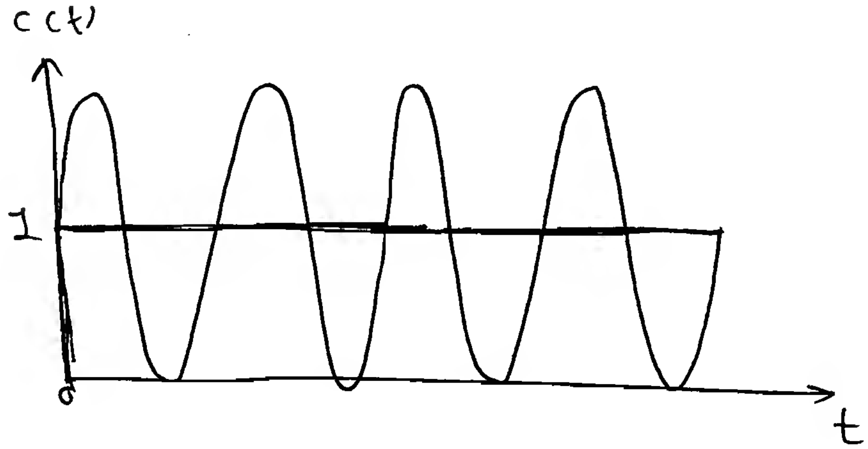
$\Rightarrow$ $\xi > 1 \rightarrow$ Overdamped system.

$\Rightarrow$ $\xi = 0$.

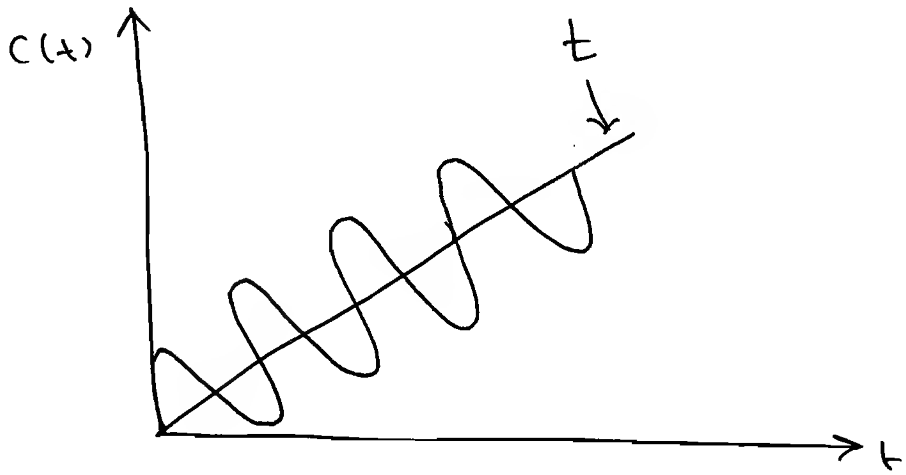
(i) Impulse



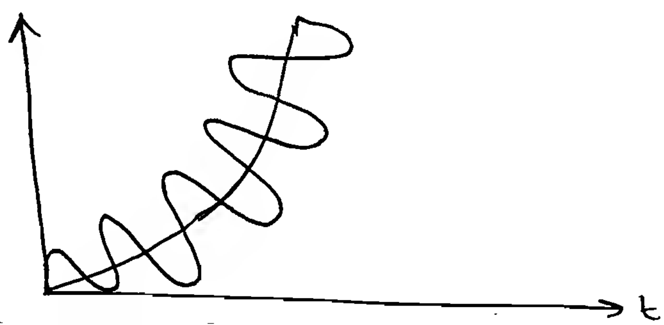
(ii) Unit step.



(iii) Unit Ramp:



(iv) Unit Parabolic:



$\Rightarrow$ when $\xi = 0$ we can not find the steady state error because the system is marginally stable.

H.B.

$\Rightarrow$ The steady state errors are calculate to only closed loop stable system.

Case - (ii) : Underdamped system $(0 \leq \xi < 1)$.

$\Rightarrow$ $s_1, s_2 = ?$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore s_1, s_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$s_1 : (s + \xi\omega_n + \omega_n\sqrt{\xi^2 - 1})$.

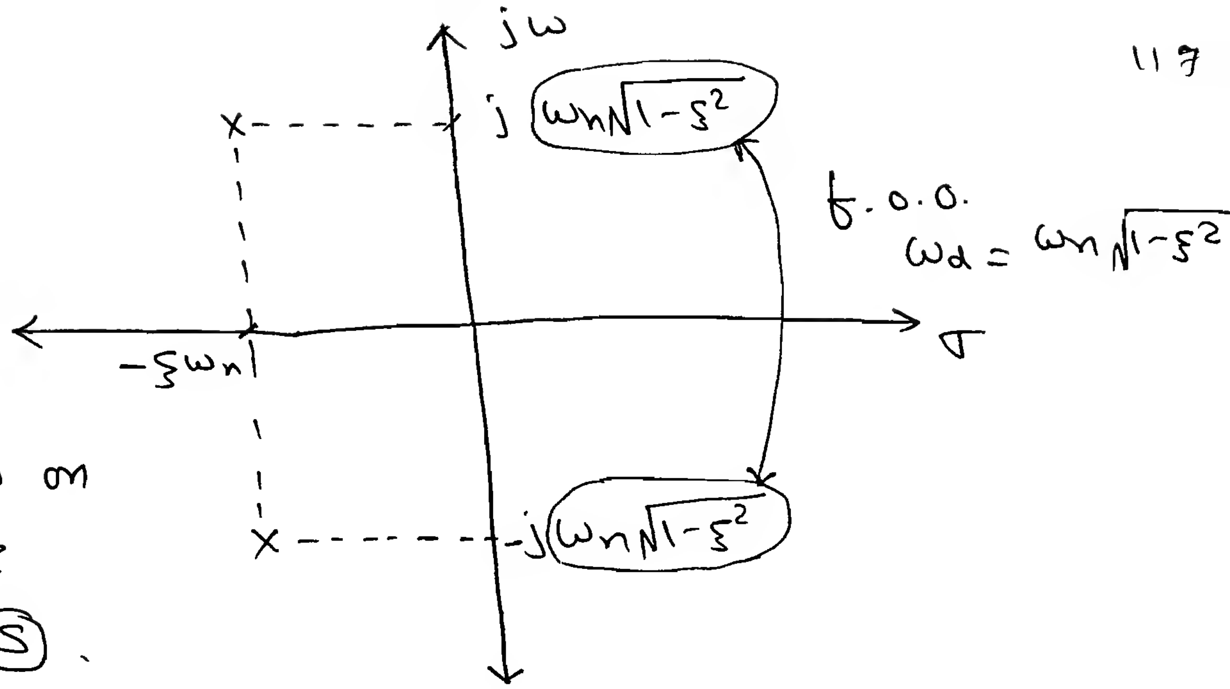
$s_2 : (s + \xi\omega_n - \omega_n\sqrt{\xi^2 - 1})$.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \xi\omega_n + \omega_n\sqrt{\xi^2 - 1})(s + \xi\omega_n - \omega_n\sqrt{\xi^2 - 1})}$$

for $0 < \xi < 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \xi\omega_n + j\omega_n\sqrt{1 - \xi^2})(s + \xi\omega_n - j\omega_n\sqrt{1 - \xi^2})}$$

$\Rightarrow$



Poles lies on left side i.e. S.

→ Real Part:

$$\boxed{\tau = \frac{1}{\xi\omega_n}}$$

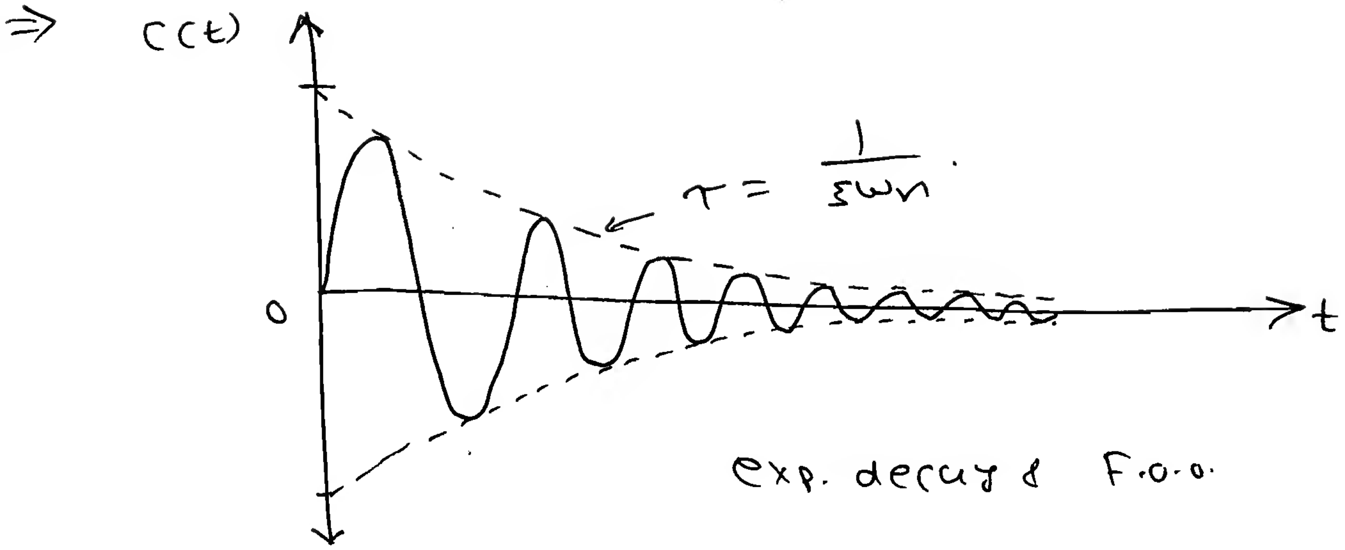
H.B.

$$\Rightarrow \boxed{\begin{array}{l} \text{Time constant } \tau = \dfrac{1}{\xi\omega_n}. \\ \text{F.o.o} = \omega_d = \omega_n\sqrt{1-\xi^2} \text{ rad/sec.} \end{array}}$$

$$C(s) = \frac{\omega_n^2}{(s+\xi\omega_n)^2 + \left(\omega_n\sqrt{1-\xi^2}\right)^2}.$$

$$\therefore C(s) = \frac{\omega_n}{\sqrt{1-\xi^2}} \cdot \frac{\omega_n\sqrt{1-\xi^2}}{(s+\xi\omega_n)^2 + \left(\omega_n\sqrt{1-\xi^2}\right)^2}.$$

$$\therefore \boxed{c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} \cdot \underbrace{e^{-\omega_n\xi t} \cdot \sin\omega_n\sqrt{1-\xi^2}\,.t}_{\text{exp. decay \& F.o.o.}}}$$



Damped oscillation } Underdamped sys.

$$\therefore \boxed{\omega_d = \omega_n \sqrt{1-\xi^2} \text{ rad/sec.}}$$

$\Rightarrow$ When $\xi > 0 \Rightarrow 0 < \xi < 1$. the poles lies in the left of s-plane which are complex conjugate. The system stable. The system response is exponential decay freq. of oscillations.

$\Rightarrow$ Any system which produced the damped oscillation is called Underdamped system.

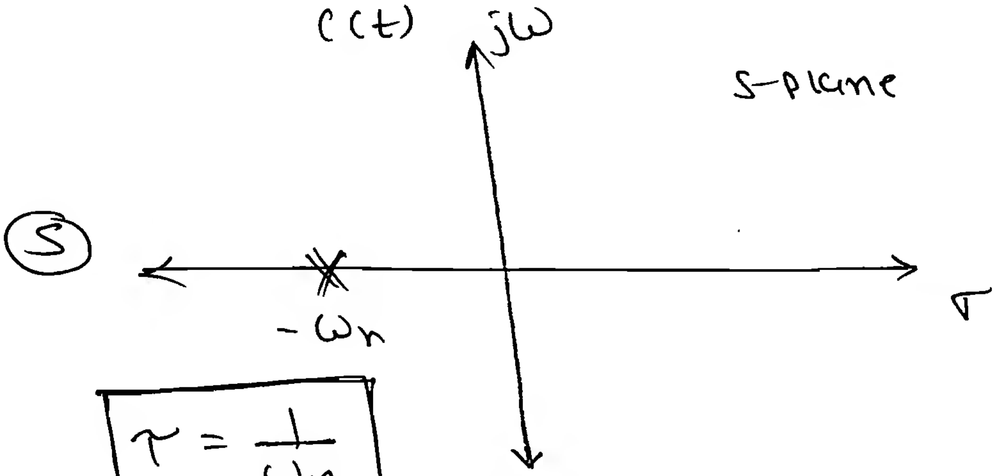
$\Rightarrow$ Case-(iii): $\xi = 1$. Critical damped:

$$\Rightarrow \frac{C(s)}{\cancel{R(s)}} = \frac{\omega_n^2}{s^2 + 2\omega_n + \omega_n^2}.$$

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{(S+\omega_n)^2}. \qquad S = -\omega_n, -\omega_n.$$

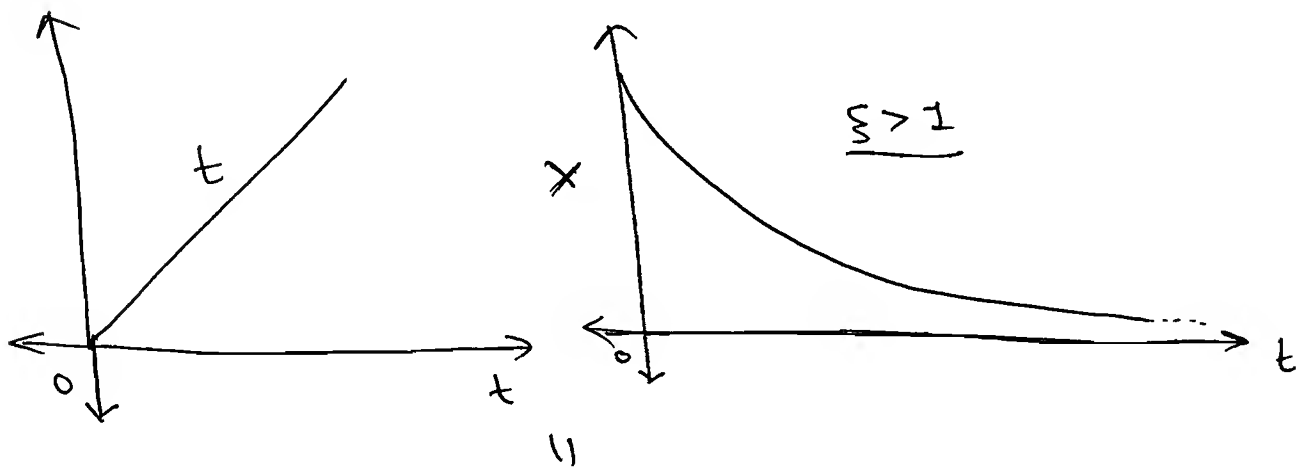
$$\therefore \boxed{c(t) = \omega_n^2 . t . e^{-\omega_n t}.}$$

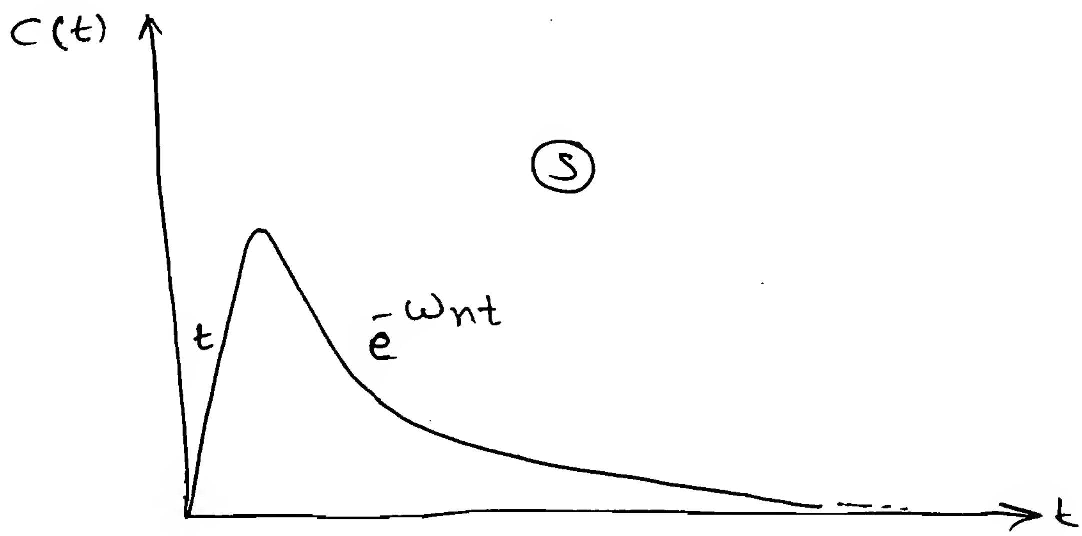
$\Rightarrow$



$$\boxed{\tau = \frac{1}{\omega_n}}$$

F.O.O. = 0.





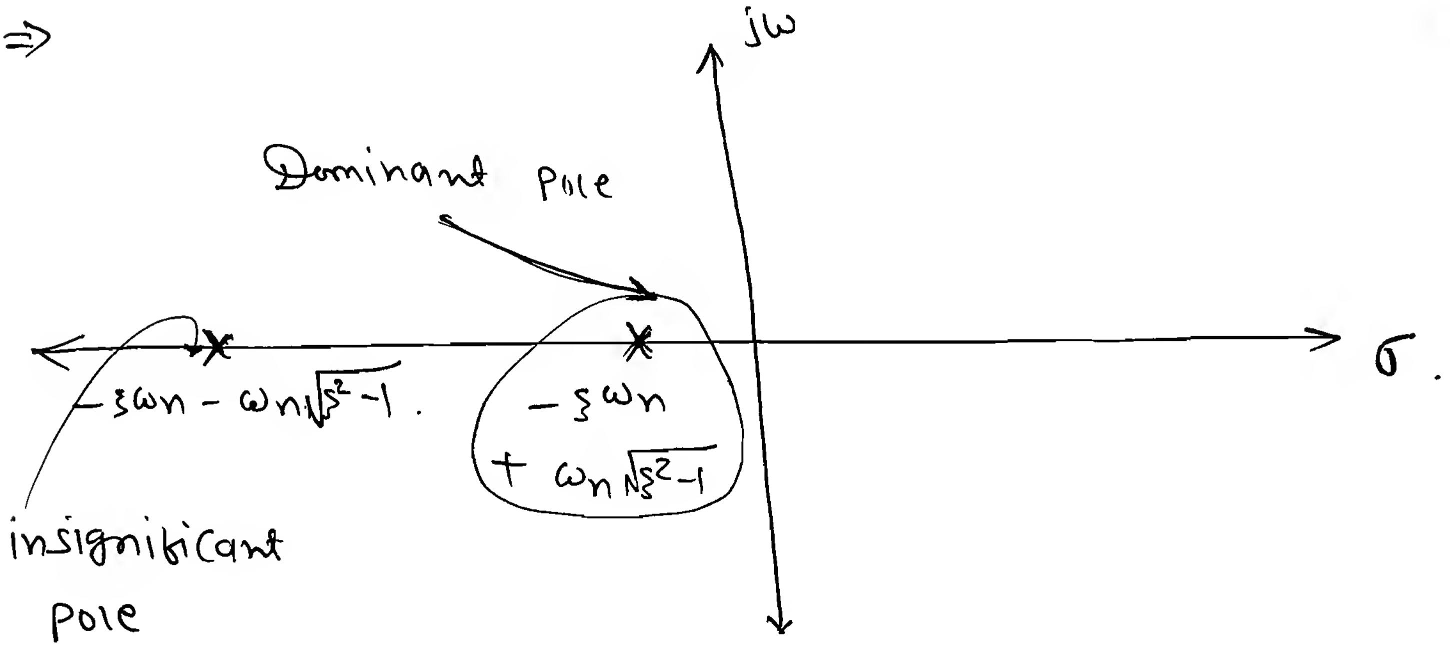
⇒ when $\xi = 1$ both the poles ~~on~~ are lies on the -ve real axis at the same location the system is stable. The system response is called as critical damped system because it generates critically one damped oscillations.

⇒ The value of Resistance used to get the critical damped nature is called critical Resistance.

Case - (iv): $\xi > 1$: Overdamped system:

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}.$$

⇒



By Real Part.

$$\boxed{\tau = \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}}$$

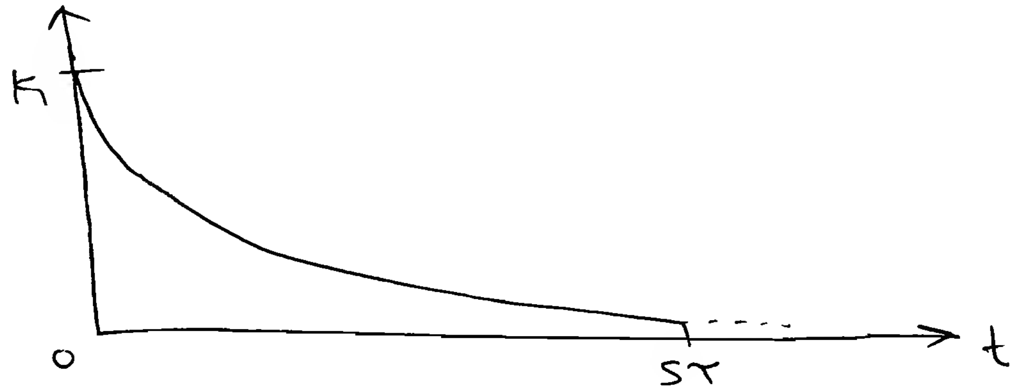
$\boxed{\text{F.} \cdot 0 \cdot 0 = 0}$

$$\Rightarrow \quad C(s) = \frac{\omega_n^2}{\left(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)\left(s+\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)}$$

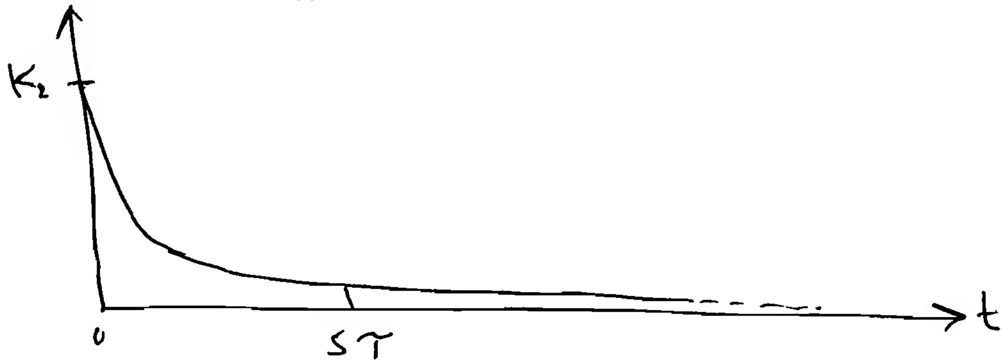
$$\therefore C(s) = \frac{K_1}{\left(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)} - \frac{K_2}{\left(s+\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)}$$

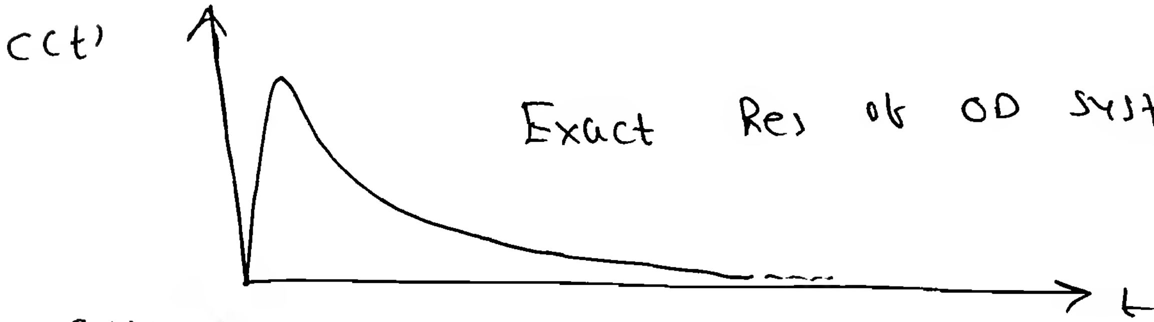
$$c(t) = \underbrace{K_1 \cdot e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}}_{\text{D.P. } (\tau\uparrow)} - \underbrace{K_2 \cdot e^{-(\xi\omega_n + \omega_n\sqrt{\xi^2-1})t}}_{(\text{I.P.})\ (\tau\downarrow)}.$$

$\Rightarrow$

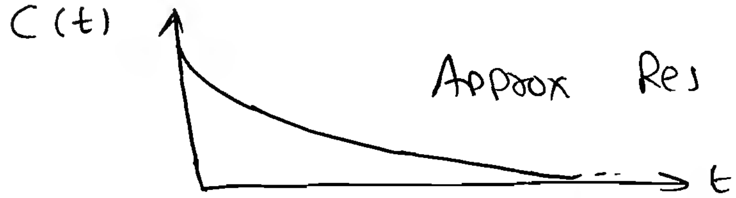


$-$





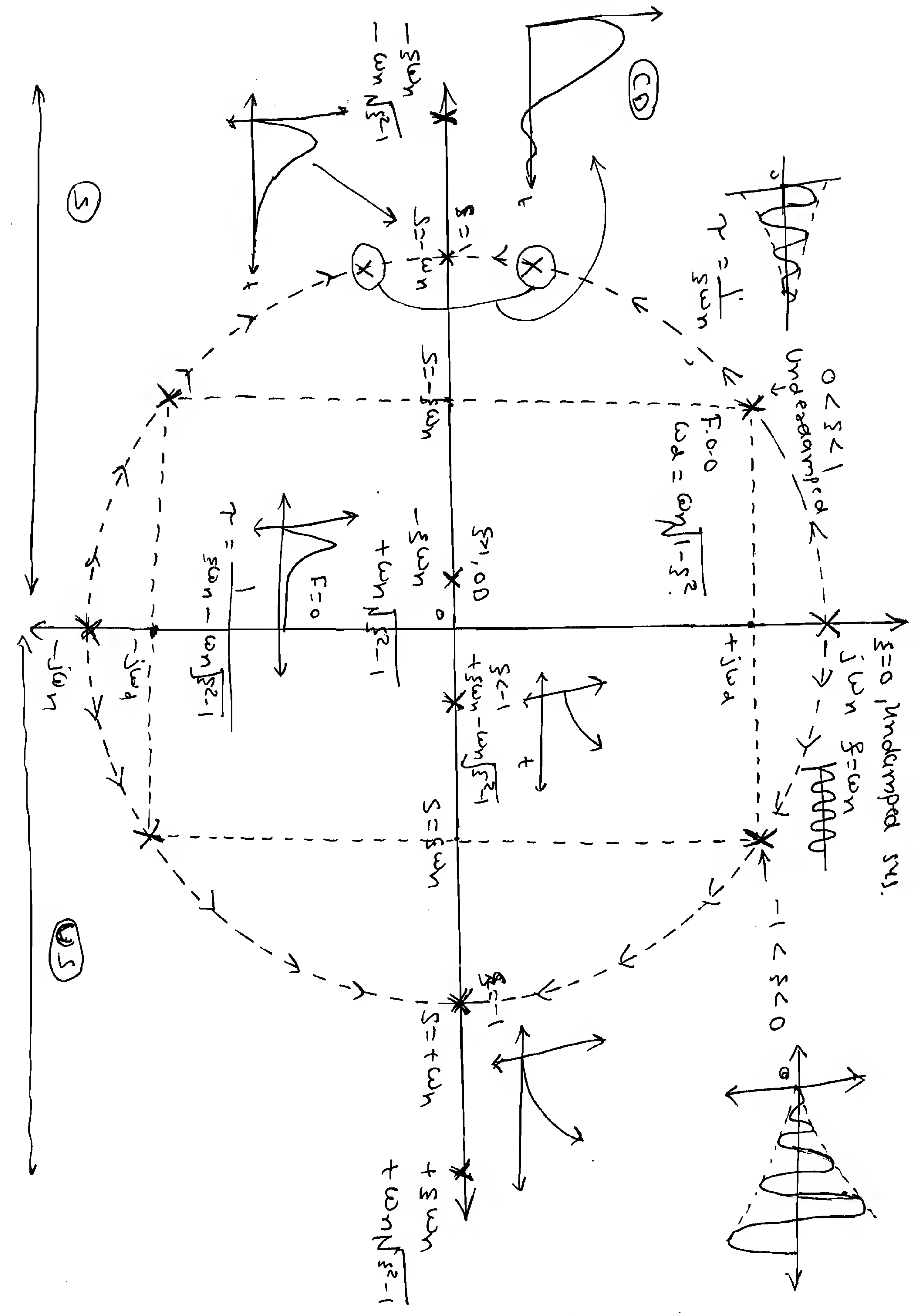
Exact Res of OD system



Approx Res of OD system.

⇒ When $\xi > 1$ both the poles lies in the left of S-plane at different location the system is stable. The system response is called over damped system because the system response elements are over comes the damped oscillation.

* Summary:

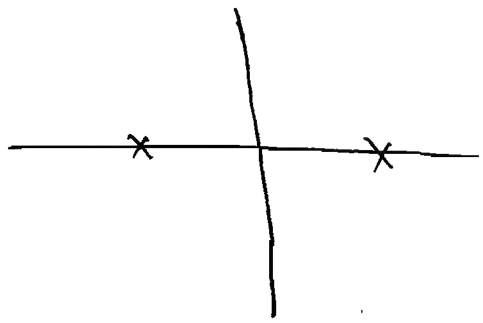

0 < ξ < 1
Underdamped
ξ = 0, Undamped sys.
-1 < ξ < 0
ξ = 1
ξ < -1
ξ = -1
j ωn
-j ωn
+j ωd
-j ωd
ωd = ωn√(1-ξ²)
S = -ξωn
S = ξωn
S = -ωn
S = +ωn
CD
US
CUS

* Conclusion:

⇒ When ξ increases from -1 to $+1$, the second order system poles path is a circle with a radius of ω_n.

⇒ Radial distance of complex pole is 'ω_n'.

⇒ The value of ξ to the given pole location is ___.



ⓐ $\xi = 1$

ⓑ $\xi = -1$

ⓒ $|\xi| = 1$ ✓

ⓓ none

⇒ When ξ increases from 0 to 1, the poles moves towards the left and near to the real-axis. In this, case

① Time constant ↓

② Satelling time ↓

③ ω_d ↓

④ as ω_d ↓ the time specification t_d, t_r, t_p ↑ and the system becomes more relatively stable.

→ when ξ increases from 1 to ∞ then one pole moves towards to the origin on the real axis. In this case.

1. τ ↑
2. t_s ↑
3. damped oscillation become 0.
   i.e. $\omega_d = 0$.
4. the relative stability of the system decreases.

⇒ <u>order of the time constant</u>.

$$\Rightarrow \tau_{undamped} > \tau_{overdamped} > \tau_{underdamped} > \tau_{critical\ damped}$$

$$\underbrace{(\infty)}_{MS} \quad \left(\frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}\right) \quad \left(\frac{1}{\xi\omega_n}\right) \quad \left(\frac{1}{\omega_n}\right)$$

S

largest τ (Slow response & sluggish system)

med τ

Smallest τ

* Unit Step response:

$\Rightarrow$ $r(t) = 1 \cdot u(t)$.

$R(s) = 1/s$.

$$\Rightarrow \quad C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

Case - (i): $\xi = 0$: Undamped System.

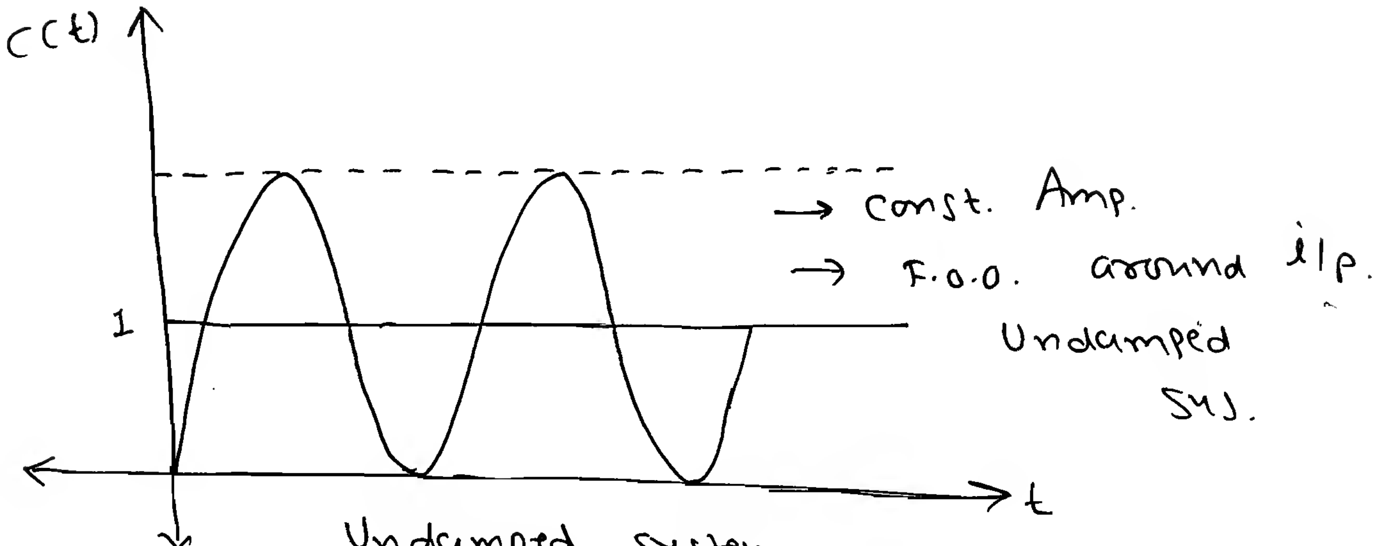
$$\rightarrow \quad C(s) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)}.$$

$$C(t) = \frac{A}{s} + \frac{Bs + C}{s^2 + \omega_n^2}.$$

$$= \frac{1}{s} + \frac{s(-1) + 0}{s^2 + \omega_n^2}.$$

$$\therefore \quad C(s) = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2}.$$

$$\therefore \quad \boxed{C(t) = 1 - \cos(\omega_n t).}$$



Undamped system

case - (iii) : $\xi > 0,\ \xi < 1$ $[0 < \xi < 1]$:

Underdamped system.

$\Rightarrow$

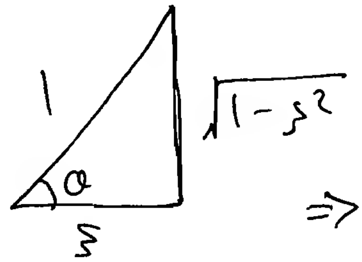
$$C(s) = \frac{\omega_n^2}{s\left(s + \xi\omega_n - j\omega_n\sqrt{1-\xi^2}\right)\left(s + \xi\omega_n + j\omega_n\sqrt{1-\xi^2}\right)}$$

$\Rightarrow$ ILT to the above eqn is,

$\Rightarrow$

$$\therefore\ \boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left[\omega_n\sqrt{1-\xi^2}\cdot t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]}$$

$\Rightarrow$ $\tan\theta = \dfrac{\sqrt{1-\xi^2}}{\xi}$



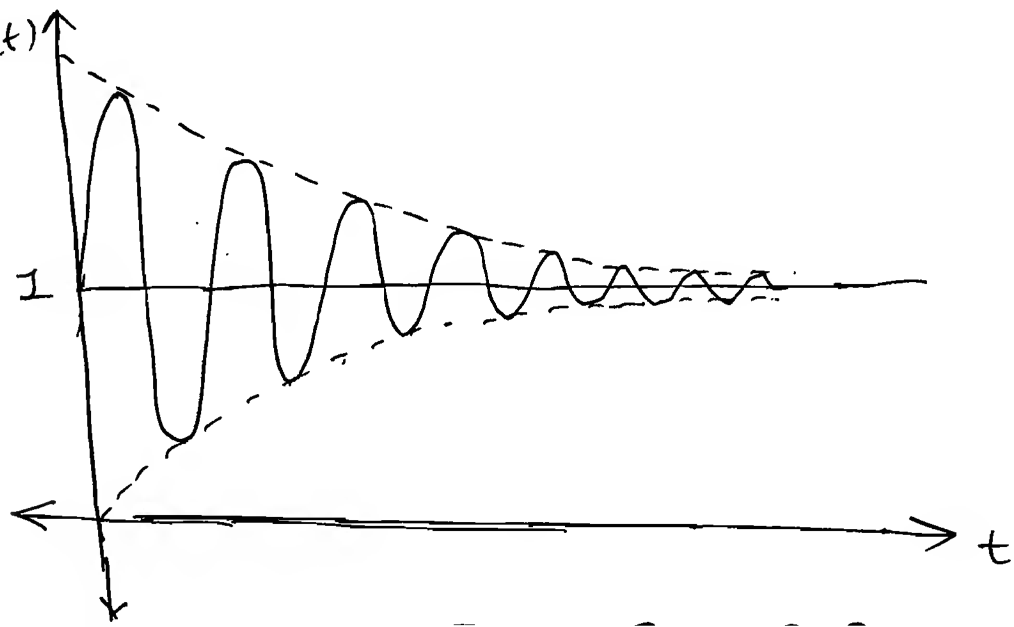
$\Rightarrow \cos\theta = \xi$

$\theta = \cos^{-1}(\xi)$.

$\Rightarrow$

$$\therefore\ \boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left(\omega_d t + \cos^{-1}(\xi)\right)}$$

$\Rightarrow$



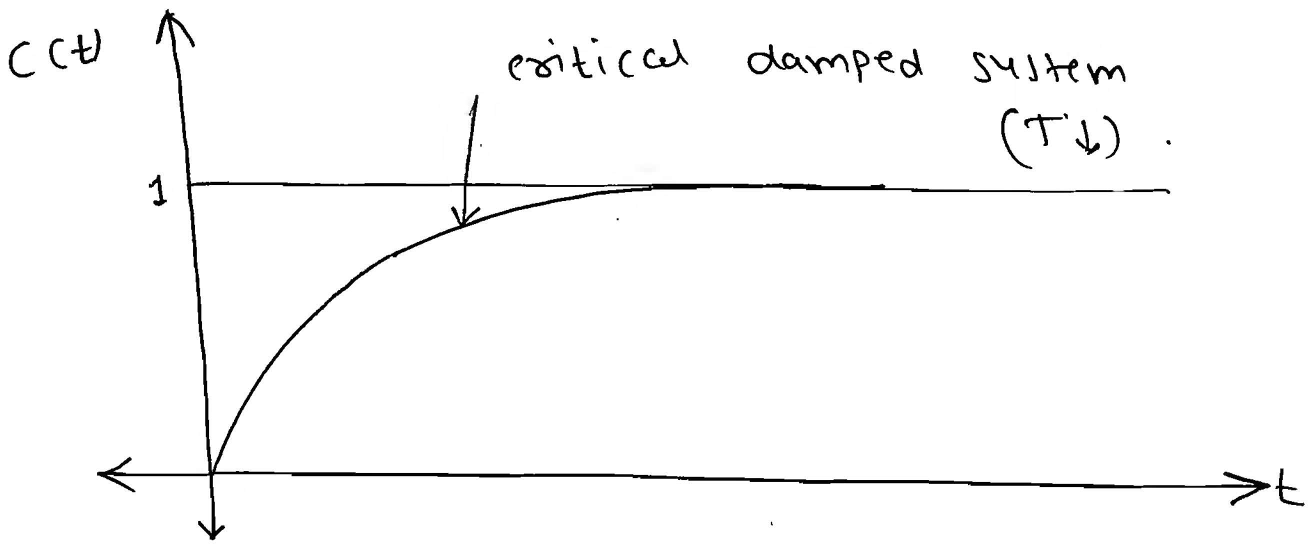
Case - III: $\xi = 1 \Rightarrow$ Critical damped.

$$\Rightarrow C(s) = \frac{\omega_n^2}{s(s^2 + 2\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{s(s+\omega_n)^2}.$$

$$= \frac{A}{s} + \frac{B}{(s+\omega_n)} + \frac{C}{(s+\omega_n)^2}.$$

$$C(s) = \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{(s+\omega_n)}.$$

$$\therefore \quad c(t) = \left(1 - \omega_n c t \cdot e^{-\omega_n t}\right) - e^{-\omega_n t}\Big).$$



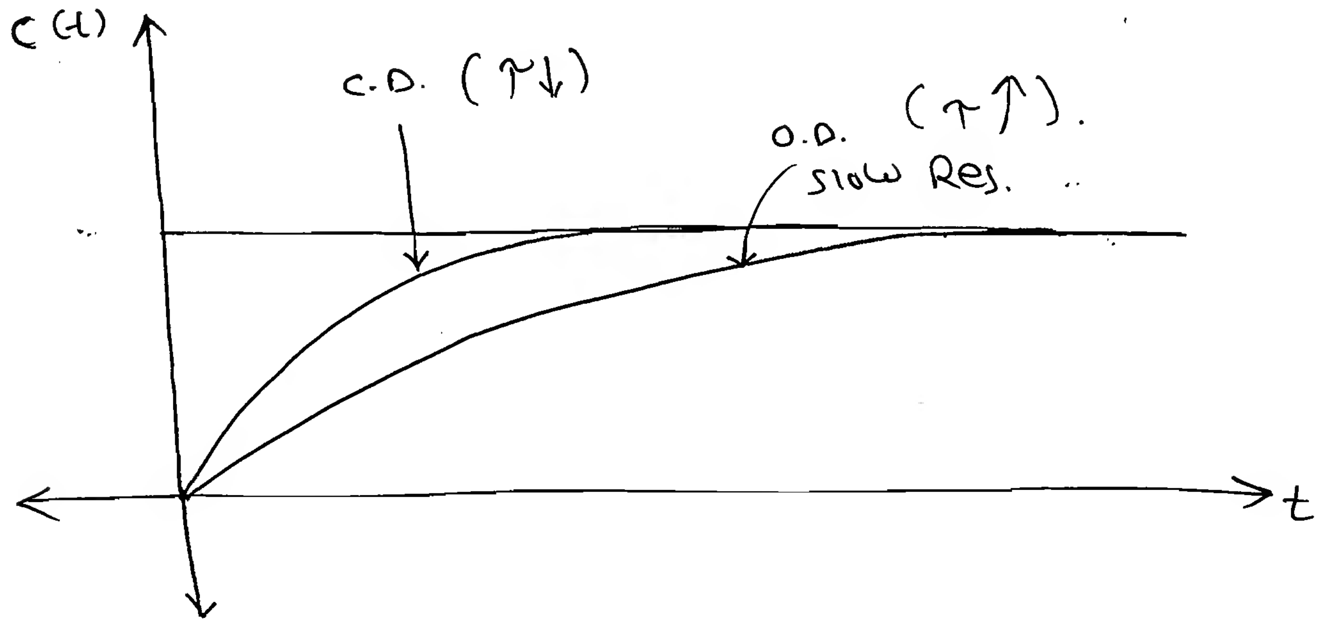
Case - IV: $\xi > 1$ overdamped.

$$\Rightarrow C(s) = \frac{\omega_n^2}{s(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1})(s+\xi\omega_n + \omega_n\sqrt{\xi^2-1})}.$$

$$= \frac{1}{s} - \frac{K_1}{(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1})} - \frac{K_2}{s+\xi\omega_n + (\omega_n\sqrt{\xi^2-1})}$$

$$\Rightarrow C(t) = 1 - K_1 \cdot e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t} - K_2 \cdot e^{-(\xi\omega_n + \omega_n\sqrt{\xi^2-1})t}$$

$$= 1 - \left[K_1 \cdot e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})} + K_2 \cdot e^{-(\xi\omega_n + \omega_n\sqrt{\xi^2-1})t}\right].$$



☆ Time Domain Specification:

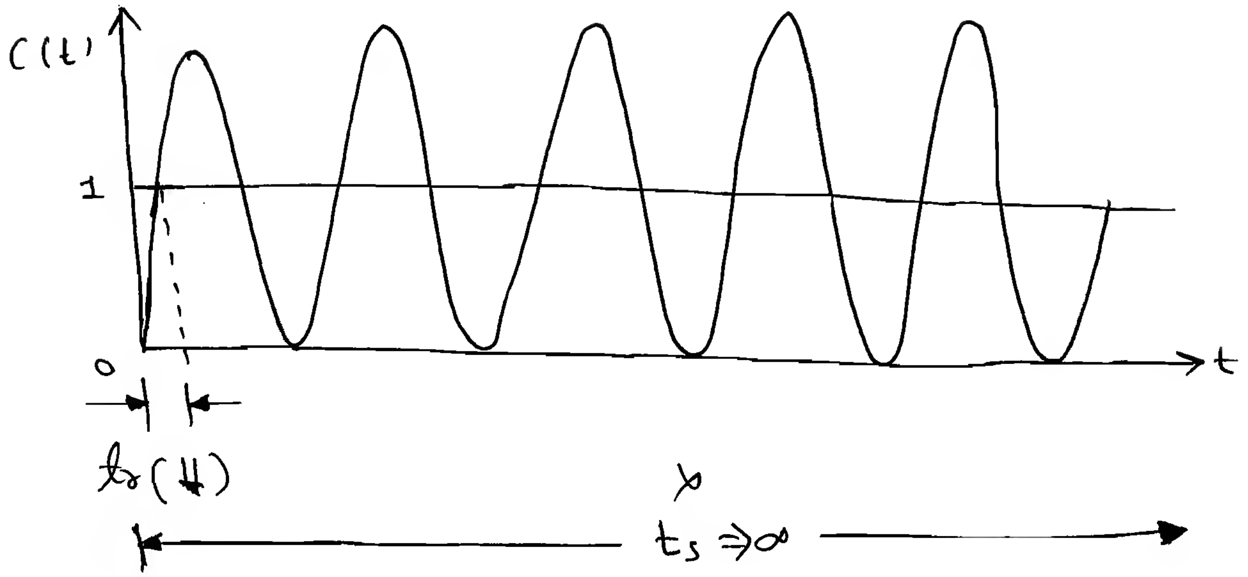
⇒ For time domain specification select a unit step input and underdamped system.

	tr.	ss.	(S)	
Impulse	✓	×	✓	← Practically not exist.
Step	✓	✓	✓	→ $\boxed{\text{Practically exist}}$
ramp	✓	✓	×	Unbounded I/p's
Parabolic	✓	✓	×.	

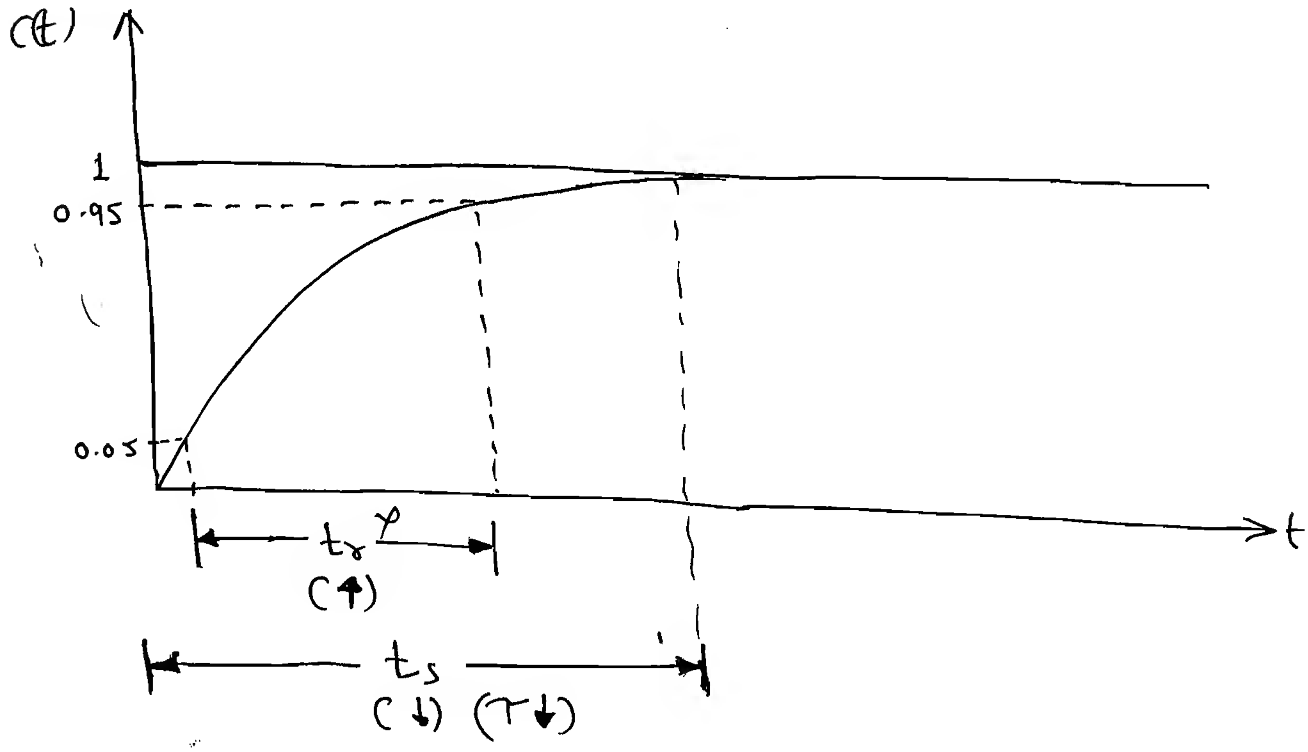
⇒ For system behaviour impulse is used.

⇒ For analysis w.r.t. time step is used.

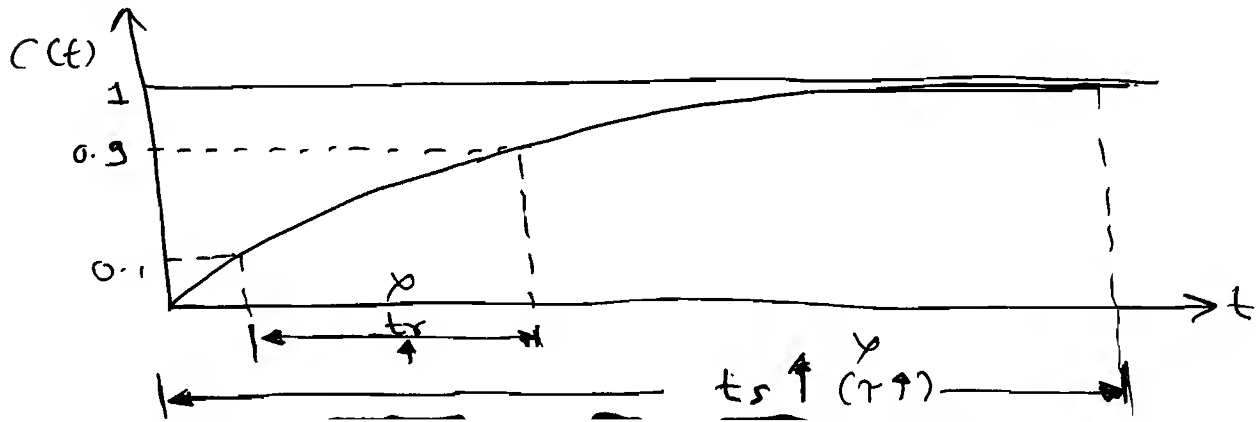
① $\xi = 0$: Undamped system:



② $\xi = 1$: Critical Damped (t_r 5% to 95%).

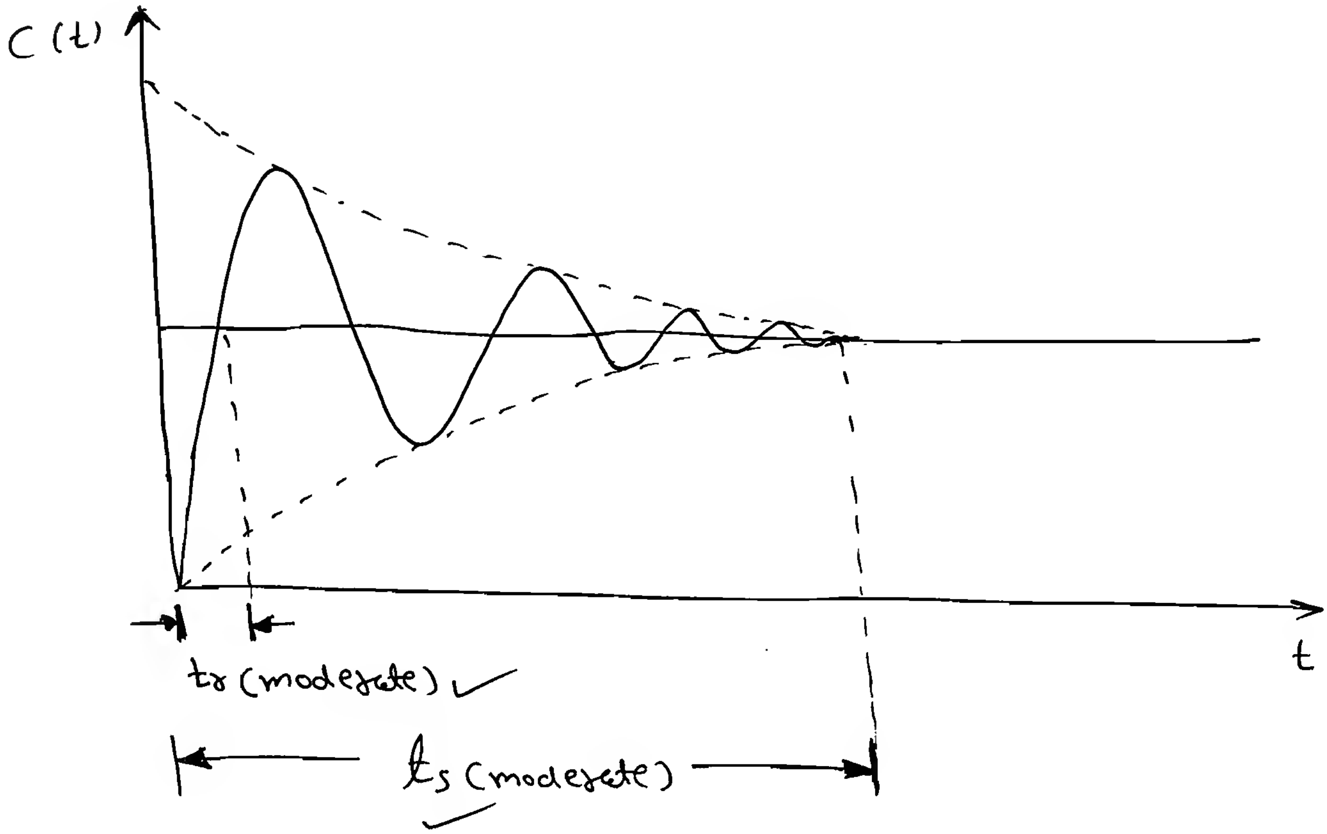


③ $\xi > 1$: overdamped (t_r : 10% to 90%).



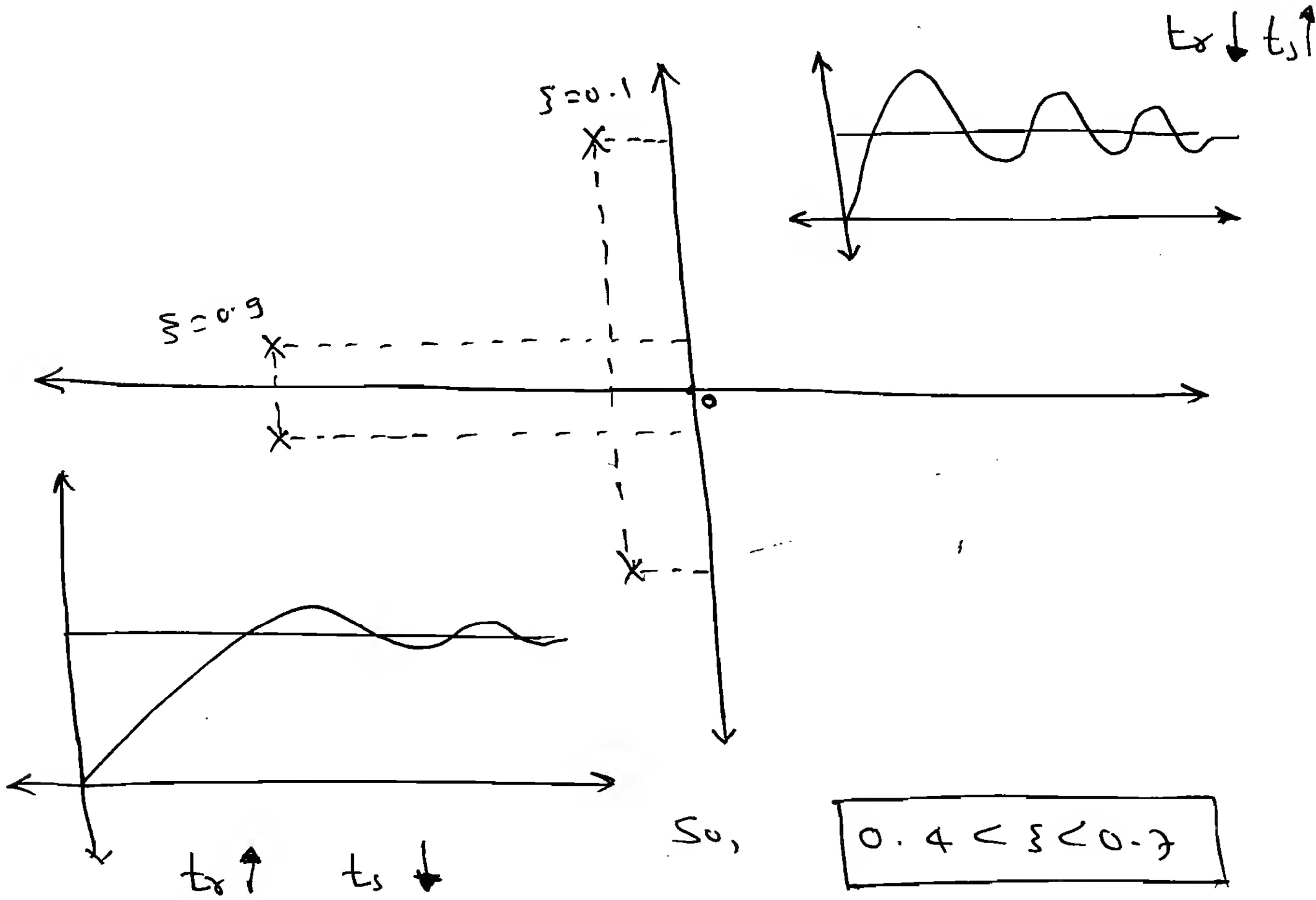
④ If $0 < \xi < 1$: Underdamped System:

or $0.4 < \xi < 0.7$.



* Optimum Value of ξ:

⇒



So, $\boxed{0.4 < \xi < 0.7}$

⇒ For time domain Specification select the underdamped system because.

① If select the undamped system the rise time is very small. The setteling time is infinity whereas if select the critical damping system rise time is large but setteling time is very small.

② If select the overdamped system the rise time is large and setting time also large.

⇒ Practically for any system, required smallest rise time and smallest setting time.

⇒ In underdamped system we can get the moderate values of rise time and setting time.

⇒ In underdamped system the best range of ξ is $0.4 < \xi < 0.7$.

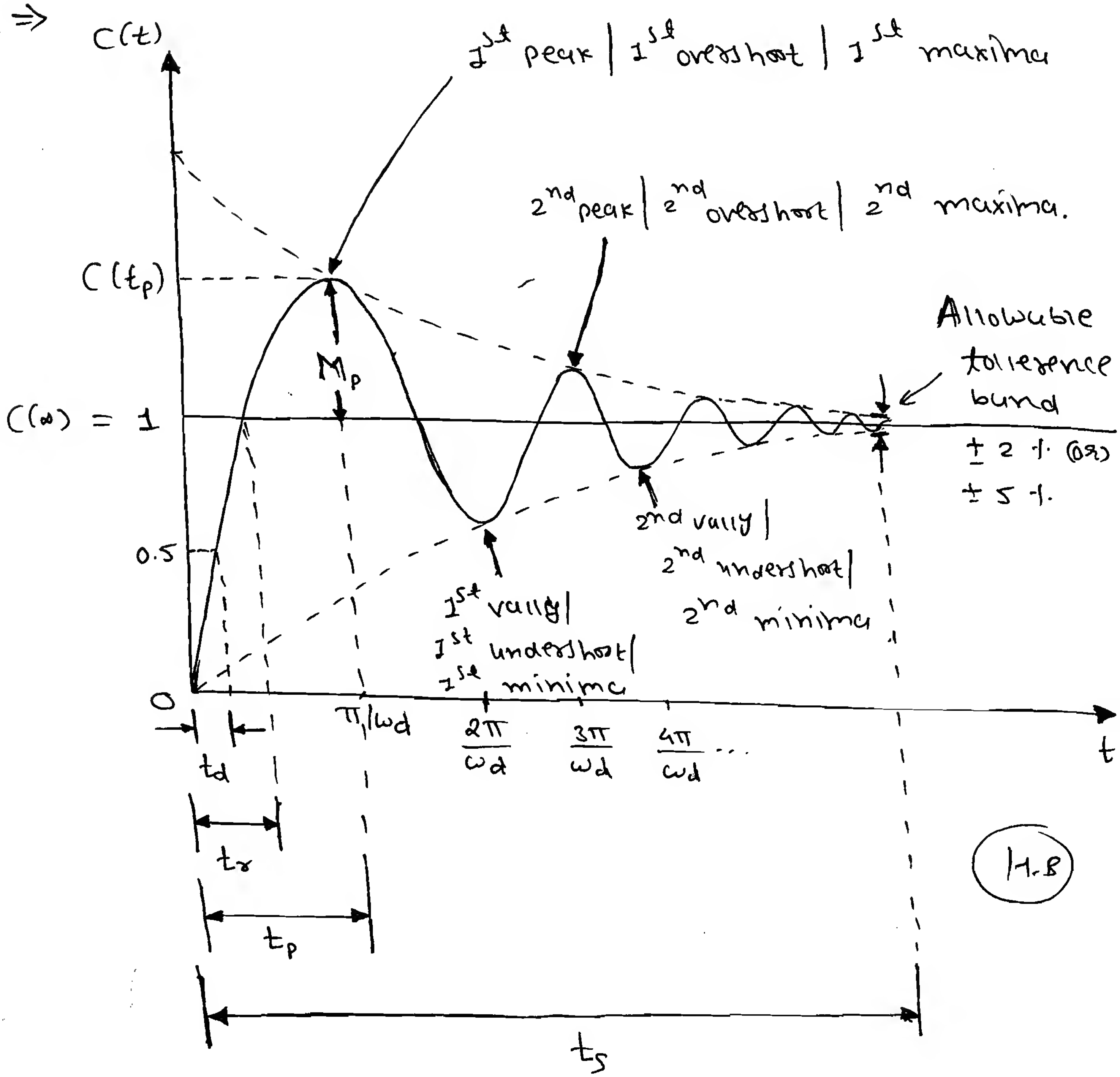
⇒ When $0 < \xi < 1$, the unit step response of the system is:

$\Rightarrow$

$$c(t) = \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left(\omega_n\sqrt{1-\xi^2} + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right].$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$\Rightarrow$



(1.8)

① Delay time :: (t_d):

$\Rightarrow$ It is the time required for the response to rise from 0 to 50 % of the final value is called the delay time.

→ denoted by t_d.

$$\boxed{t_d = \frac{1 + 0.75}{\omega_n} \text{ sec.}}$$ ⇐ H.B.

② Rise time (t_r):

⇒ It is the time required for the response to rise from 0 to 100 % of its final value is ensured for underdamped system, 5 % to 95 % for critical damped system and 10 % to 90 % for over damped system.

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d}$$

$$\therefore \boxed{t_r = \frac{\pi - \cos^{-1}(\xi)}{\omega_d}}$$

③ Peak - time (t_p):

⇒ It is the time required for the system to rise from 0 to peaks of the time response.

$\Rightarrow$ $$\boxed{t_p = \frac{n\pi}{\omega_d}}$$

$n = 1 \rightarrow$ by default $\rightarrow$ 1st peak.

$$\therefore \boxed{t_p = \frac{\pi}{\omega_d}}$$

$\rightarrow$ For 2nd peak,

$$\boxed{t_p = \frac{3\pi}{\omega_d}}$$

$\Rightarrow$ For 1st vally, $\boxed{t_p = \frac{2\pi}{\omega_d}}$

④ Peak- overshoot (M_p):-

$\Rightarrow$ It is the difference betn the time response peak to the steady state vame.

$$\boxed{M_p = c(t_p) - c(\infty)}$$

⑤ % of Peak - overshoot (% M_p):

$\Rightarrow$ It is the normalized difference betn time response peak to steady state.

$$\% M_p = \frac{c(t_p) - \cancel{c(\infty)}^{1}}{\cancel{c(\infty)}^{1}} \times 100\%$$

$$\therefore \% M_p = [c(t_p) - 1] \times 100\%$$

$$\Rightarrow \boxed{\% M_p = e^{-n\pi\xi/\sqrt{1-\xi^2}} \times 100\,\%}$$

$n=1 \Rightarrow$ by defaul $\rightarrow$ 1st peak

$$\Rightarrow \boxed{\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\,\%}$$

⇒ The n- value is similar to peak time.

⇒ The undershoot to the first vally point is.

$$\boxed{\% M_v = e^{-2\pi\xi/\sqrt{1-\xi^2}} \times 100\,\%}$$

(6) Setteling time (T_s):-

⇒ It is the time required for the response to rise from 0 to specified tolerance band usually ± 2 % (or) 5 %.

$\Rightarrow \pm 5\,\%$, $\quad t_s = 3\tau = \dfrac{3}{\xi\omega_n}$ sec.

$\pm 2\,\%$, $\quad \boxed{t_s = 4\tau = \dfrac{4}{\xi\omega_n}\text{ sec.}} \rightarrow$ default.

$\pm 0\,\%$, $\quad t_s = 5\tau = \dfrac{5}{\xi\omega_n}$ sec. (SS value).

(7) Time Period of Oscillation:

⇒ It is the time required to complete one cycle,

$$\boxed{\tau_{osc} = \frac{2\pi}{\omega_d} = 2\,T_p.}$$

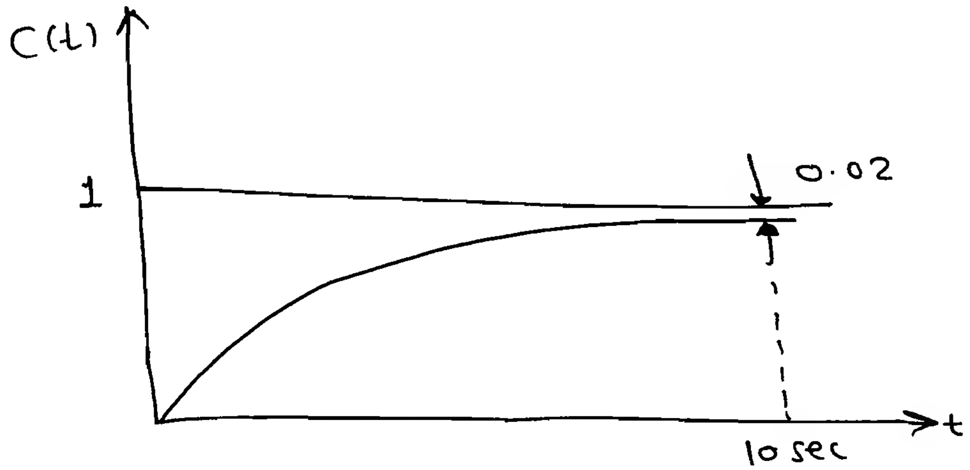
⇒ No. of oscillation before reaching Steady State.

$$N = \frac{t_s \ (\pm 2\%.\ (or)\ \pm 5\%).}{\tau_{osc}.}$$

$$\therefore \quad N = \frac{t_s}{2\pi/\omega_d} = \frac{t_s \cdot \omega_d}{2\pi}.$$

$$\therefore \quad \boxed{N = \frac{t_s}{2\,t_p}}.$$

Q The unit step response of the system is shown in fig. ~~and~~ Find the following factors. (1) τ (2) t_d (3) t_r (4) t_p & M_p.



Soln: t_s = 10 sec

$t_s = \frac{4\tau}{1}.$ (∵ ± 2%).

∴ $t_s = 4\tau$ ⇒ $\tau = t_s/4$

$$\boxed{\tau = 10/4 = 2.5\ sec.}$$

⇒ The standard form of the unit step response is,

$$\boxed{c(t) = k(1 - e^{-t/\tau}).}$$

k → s.s. value

$\therefore \quad c(t) = (1 - e^{-t/\tau})$

$\therefore \quad c(t) = (1 - e^{-t/2.5}).$

① $\tau = 2.5$ sec.

② t_d.

→ at $t = t_d \Rightarrow c(t) = 0.5$

$\therefore \quad 0.5 = 1 - e^{-t_d/2.5}.$

$\therefore \quad e^{-t_d/2.5} = 0.5$

$\therefore \quad \boxed{t_d = 1.733 \text{ s}}$

③ t_r

→ ~~at $t = t_r \Rightarrow c(t) = 1.$~~

→ For rise time consider the time duration from 10 % to 90 % of the final value,

at $t = t_{r1} \Rightarrow c(t) = 0.1.$

$\therefore \quad 0.1 = 1 - e^{-t_{r1}/2.5} \Rightarrow \boxed{t_{r1} = 0.263}$

at $t = t_{r2} \Rightarrow c(t) = 0.9.$

$\therefore \quad 0.9 = 1 - e^{-t_{r2}/2.5} \Rightarrow \boxed{t_{r2} = 5.76\,s}$

$\therefore \quad t_r = t_{r2} - t_{r1} = 2.2\tau.$

$\therefore \quad \boxed{t_r = 2.2\tau}$

$\therefore \quad t_r = 2.2 \times 2.5$

$\boxed{t_r = 5.5 \text{ sec}}$

⇒ (4) t_p & M_p.

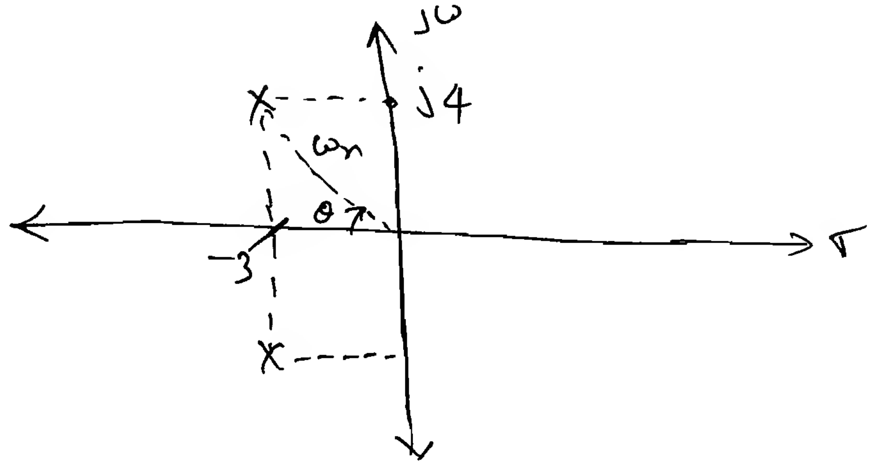
→ there is no peaks are exist hence no. peak time and no peak overshoot.

Q The impulse response of a system is $c(t) = k \cdot e^{-3t} \cdot \sin 4t$. Find the following factors:

(1) t_s (2) ω_n (3) ζ (4) M_p (5) t_d (6) t_r (7) t_p.

Soln:

$c(t) = k \cdot e^{-\overset{1/T}{3}t} \cdot \sin \overset{\omega_d}{4}t$



$\Rightarrow$ ① $\tau = 1/3$ sec.

$\omega_d = 4$ rad/sec.

$\omega_n = \sqrt{3^2 + 4^2}$

$\Rightarrow$ $\boxed{\omega_n = 5 \text{ rad/sec.}}$

② $\boxed{\omega_n = 5 \text{ rad/sec}}$

③ $\underline{\underline{\xi}}$:

$\therefore \quad \omega_d = \omega_n \sqrt{1-\xi^2}$. (or)

$\therefore \quad \left(\frac{4}{5}\right)^2 = 1 - \xi^2$

$\therefore \quad 1 - \frac{16}{25} = \xi^2$

$\xi = 3/5$

$\boxed{\xi = 0.6}$

$\xi \neq \ell$

$\theta = \cos^{-1}\xi$

$\xi = \cos\theta$

$\xi = \cos\left[\tan^{-1}\left(\frac{4}{3}\right)\right]$

$\boxed{\xi = 0.6}$

④ $\underline{\underline{M_p}}$:

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 \%$$

$$= e^{-\frac{3.14 \times 0.6}{\sqrt{1-0.36}}} \times 100 \%$$

$\Rightarrow$ $\boxed{\% M_p = 9.5 \%}$

⑤ $\underline{\underline{t_d}}$:

$$t_d = \frac{1 + 0.7\xi}{\omega_n}.$$

$$\Rightarrow \quad t_d = \frac{1 + (0.7)(0.6)}{5}$$

$$\boxed{t_d = 0.284 \text{ sec}}$$

⑥ $\underline{t_r}$:

$$t_r = \frac{\pi - \cos^{-1}(\xi)}{\omega_d}$$

$$= \frac{3.14 - \cos^{-1}(0.6)}{\omega_d}$$

$$= \frac{3.14 - 53.13^\circ}{4}$$

$$= \frac{3.14 - \left(53.13^\circ \times \frac{\pi}{180^\circ}\right)}{4}$$

$$\therefore \boxed{t_r = 0.553 \text{ sec}}$$

⑦ $\underline{t_p}$:

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4}$$

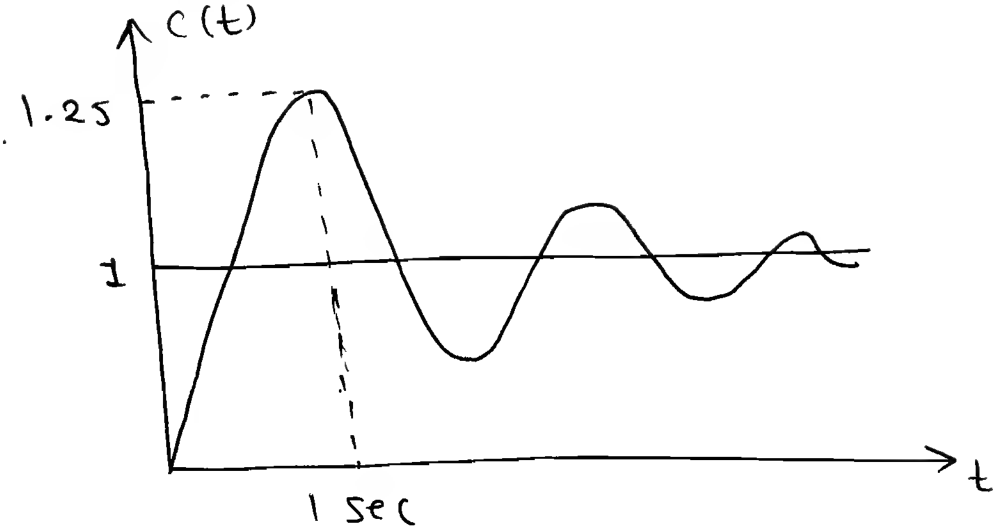
$$\therefore \boxed{t_p = 0.785 \text{ sec.}}$$

$\boxed{Q}$ The unit step response of the system is shown in figure. Find the following factor.

① M_p
② $\% M_p$
③ ξ
④ ω_n
⑤ delay time.
⑥ t_r
⑦ t_s
⑧ OLTF
⑨ CLTF.

assume UFB System.

→



soln:

① M_p:

$c(t_p) = 1.25 \quad , \quad c(\infty) = 1$

$\therefore \; M_p = c(t_p) - c(\infty) = 1.25 - 1 = 0.25$

$\therefore \; \boxed{M_p = 0.25}$

② $\% M_p$:

$$\therefore \quad \% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100 \, \%$$

$$\boxed{\% M_p = 25 \, \%}$$

③ ξ:

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 .$$

$$\therefore \quad \frac{25}{100} = e^{-\pi\xi/\sqrt{1-\xi^2}}.$$

$$\therefore \quad \frac{-\pi\xi}{\sqrt{1-\xi^2}} = -1.386$$

$$\therefore \quad \frac{\zeta}{\sqrt{1-\zeta^2}} = 0.441$$

$$\therefore \quad \zeta^2 = (0.195)\,(1-\zeta^2).$$

$$\therefore \quad 1.195\,\zeta^2 = 0.195$$

$$\Rightarrow \boxed{\zeta = 0.404.}$$

④ $\underline{\underline{\omega_n}}$:

$$\cancel{t_s = 4\,sec.}$$

$$\cancel{t_s = 4T.}$$

$$\cancel{t_s = \frac{4}{\zeta\omega_n}} \Rightarrow \cancel{\omega_n = \frac{4}{\zeta t_s} = \frac{4}{0.4}}$$

$$\therefore \quad t_p = \frac{\pi}{\omega_d}.$$

$$\therefore \quad \omega_d = 3.14.$$

$$\Rightarrow \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\therefore \quad \omega_n = \frac{3.14}{\sqrt{1-0.16}}$$

$$\Rightarrow \boxed{\omega_n = 3.43 \ rad/sec}$$

⑤ $\underline{\underline{t_d}}$:

$$\therefore \quad t_d = \frac{1 + 0.7\zeta}{\omega_n}.$$

$$= \frac{1 + (0.7)(0.4)}{3.43}$$

$$\therefore \boxed{t_d = 0.373 \ sec}$$

⑥ $\underline{\underline{t_r}}$:

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

$$= \frac{3.14 - \cos^{-1}(0.4)}{3.14}$$

$$\therefore \boxed{t_r = 0.63 \text{ sec}}$$

⑦ $\underline{\underline{t_s}}$:

$$t_s = 4\tau = \frac{4}{\xi\omega_n}$$

$$\therefore t_s = \frac{4}{0.4 \times 3.43}$$

$$\therefore \boxed{t_s = 2.915 \text{ sec}}$$

⑧ <u>OLTF</u>:

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}$$

$$= \frac{(3.43)^2}{s(s + 2(0.4)(3.43))}$$

$$\boxed{G(s) = \frac{11.765}{s(s + 2.744)}}$$

⑨ <u>CLTF</u>:

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore \boxed{\frac{C(s)}{R(s)} = \frac{11.765}{s^2 + 2.744s + 11.765}}$$

$\boxed{Q}$ Find the $\% M_p$ to the following systems to the unit step input.

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+25}.$$

Solⁿ: Here, $\omega_n = 5$ rad/sec

but $\xi = 0$.

So, $\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \cdot 100\,\%$.

$\% M_p = e^{-\pi \cdot 0/\sqrt{1-0}} \cdot 100\,\%$.

$\therefore \boxed{\% M_p = 100\,\%}$

$\boxed{Q}$ Find M_p of $\frac{C(s)}{R(s)} = \frac{100}{s^2+20s+100}$.

Solⁿ: Here, $\omega_n^2 = 100$.

$\Rightarrow \boxed{\omega_n = 10 \text{ rad/sec}}$

$2\xi\omega_n = 20$ 10

$\boxed{\xi = 1} \rightarrow \boxed{CD}$

$\therefore M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\,\%$.

$= e^{-\pi\xi/\sqrt{1-1}} \times 100\,\%$.

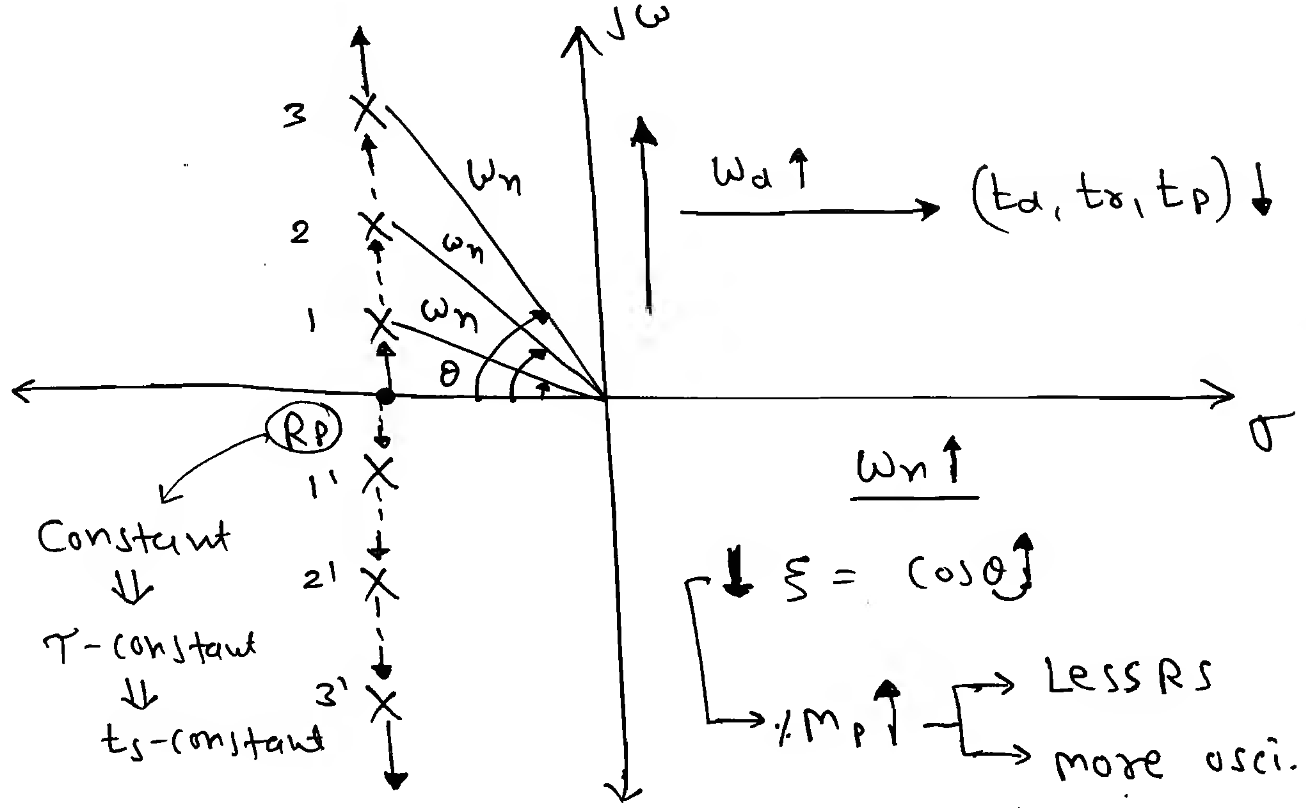
$M_p = \cancel{100}\ 0\,\%$. $\boxed{M_p = 0\,\%}$.

Note: When ξ increases from 0 to 1, % M_p is decreases from 100 % to 0 %.

→ When $\xi \geq 1$, % $M_p = 0$ %. because no ~~the~~ oscillation are exist in the system.

$\boxed{Q}$ Find the Variation in time domain specification to the given Poles Path in the S-plane.

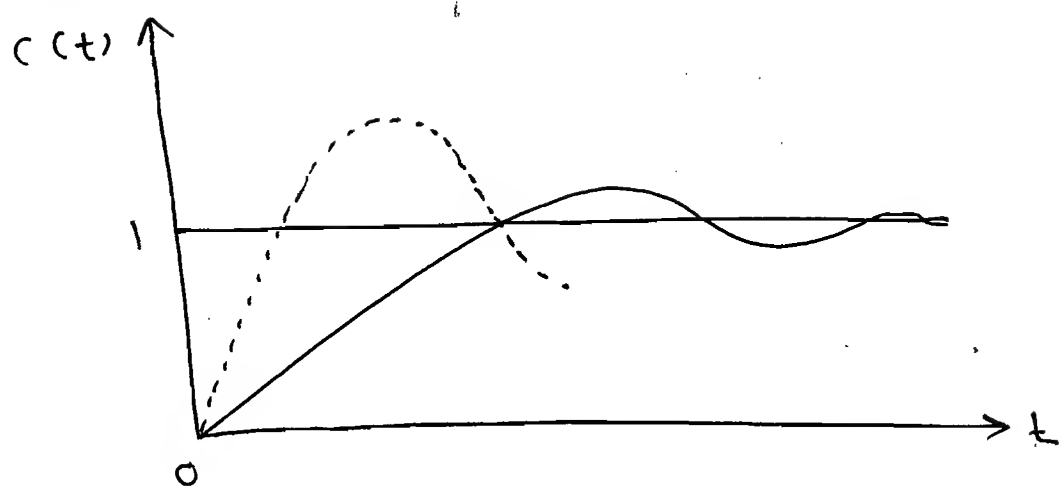
Soln.



⇒ As real part is constant, time constant is constant hence setteiing time is constant.

⇒ As imaginary part increases the damped oscillation ω_d increases. As ω_d

increases the time specification t_d, t_r & t_p decreases.

=> 

$\boxed{\text{Optimum value of } \% M_p = 5\% \text{ to } 25\%}$

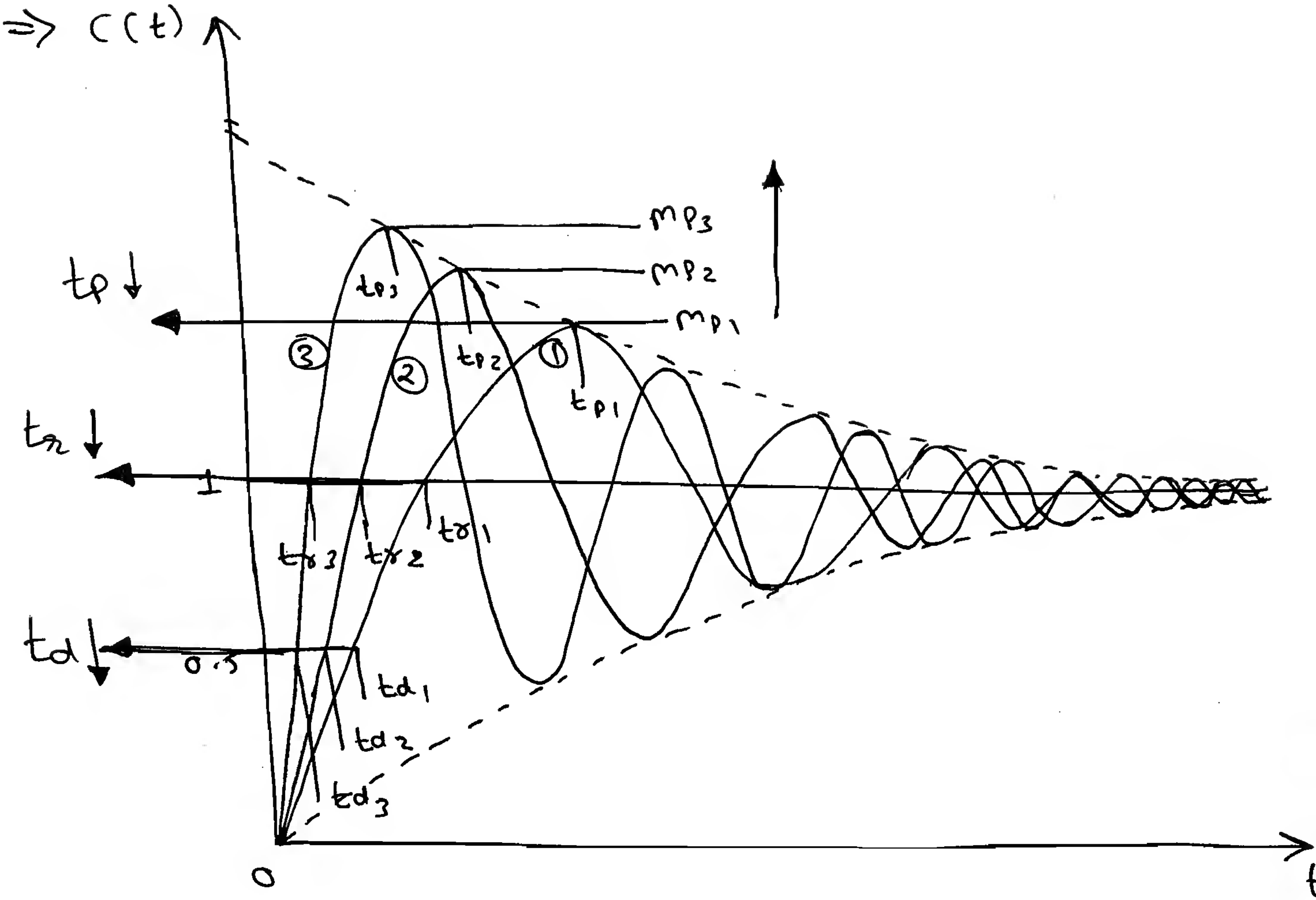
=> As the inclination of the pole θ increases the damping ratio ξ decreases. Hence, the % of M_p increases.

=> The large M_p make the system less RS & more oscillatory.

=> The Optimum range of the % M_p is 5% to 25%.

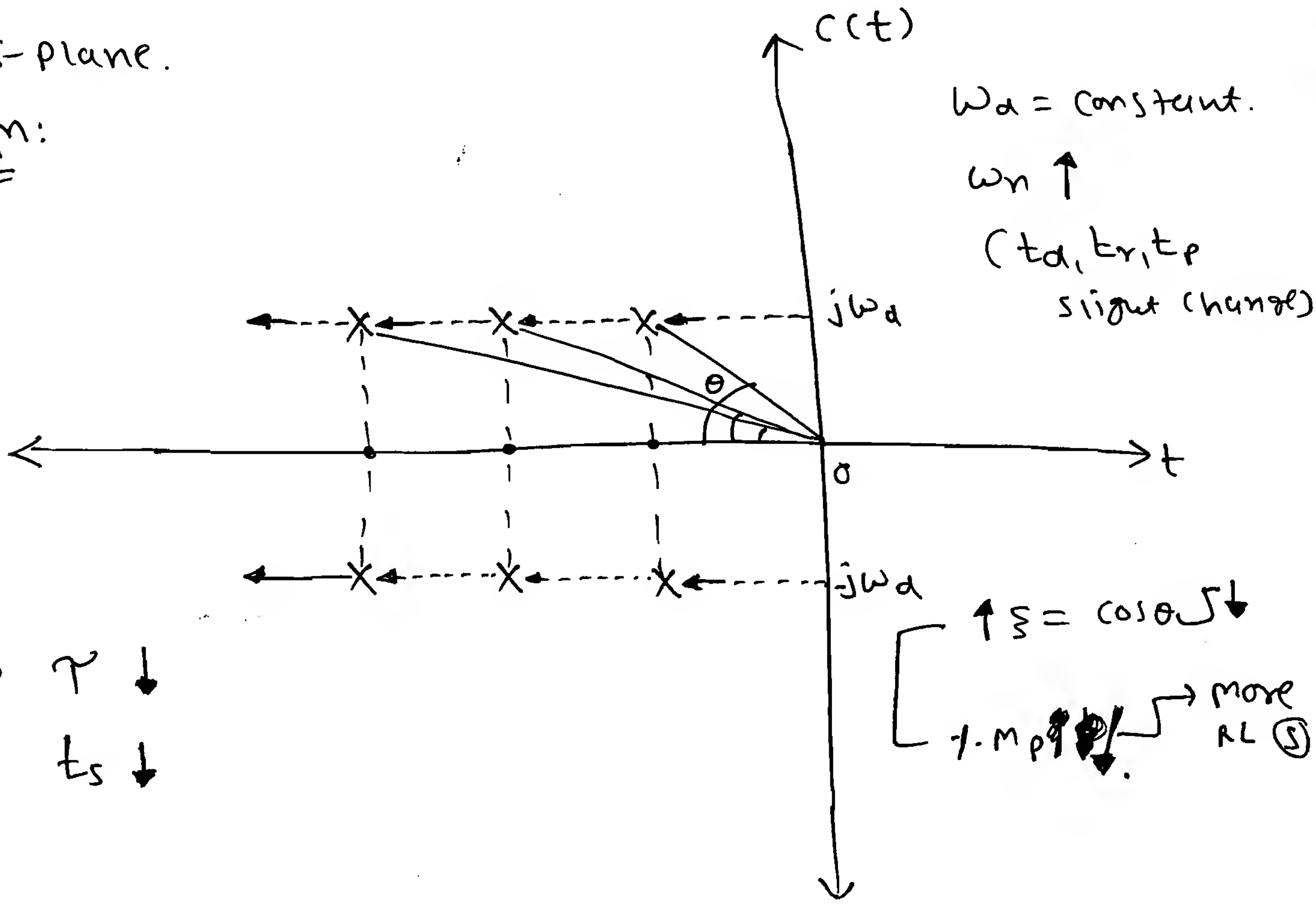
=> If the peak overshoot is more than 25% the system is less relative stable.

=> If the peak overshoot is $M_p < 5\%$ the system is slow response.



[a] Find the Variation in time domain Specification to the given Poles path in S-plane.

Soln:



$\Rightarrow \tau \downarrow$

$t_s \downarrow$

=> Pole moving towards the left side and t_s & τ both decreases.

=> Imaginary part is constant.

so, $\omega_d =$ constant.

$$t_p = \frac{\pi}{\omega_d} = \text{Constant.}$$

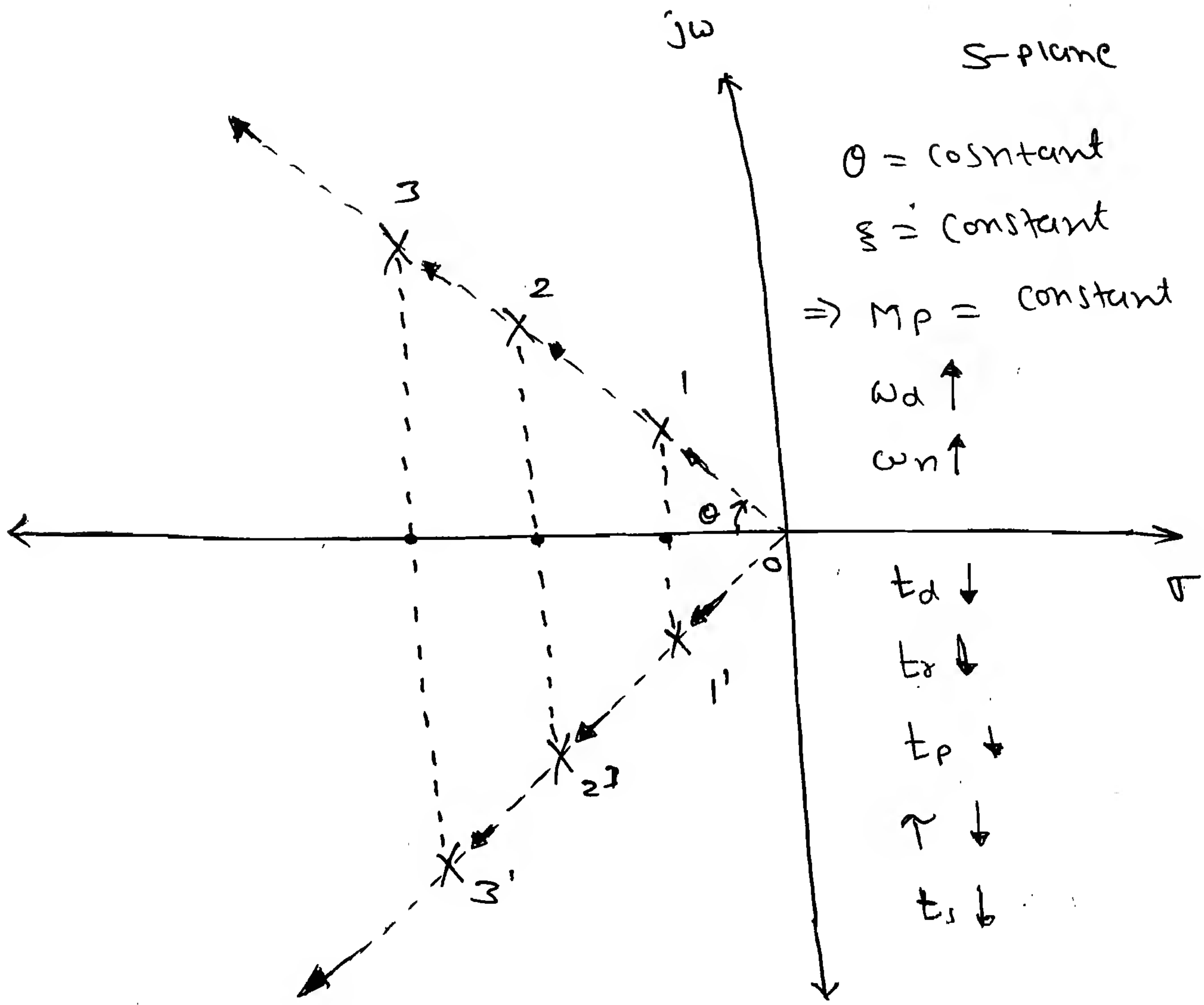
=> $t_p =$ constant.

=> As imaginary part is constant the damped oscillations ω_d constant but there exist a slight variation in t_d and t_r.

=> As the inclination at the pole 'θ' decreases the damping ratio ξ increases. Hence the % M_p ~~increases~~ decreases. The system become more relative stable.

Q Find the variation in the time specification when location of poles moves (or) change as shown in fig.

⇒



θ = constant

ξ = constant

⇒ M_p = constant

$\omega_d \uparrow$

$\omega_n \uparrow$

$t_d \downarrow$

$t_r \downarrow$

$t_p \downarrow$

$\tau \downarrow$

$t_s \downarrow$

⇒ As the inclination of the pole θ = constant the damping ratio ξ is constant and hence % M_p is also constant.

⇒ As the poles moves towards the left time constant τ decreases hence settelling time decreases.

⇒ As imaginary part is increases damped oscillation ω_d increases hence $(t_d, t_r, t_p)\downarrow$ hence $\omega_n \downarrow$.

$\boxed{Q}$ Find the time domain specification of 151

$$G(s) = \frac{25}{s(s+4)}, \quad H(s) = 1.$$

Soln:

$$G(s) = \frac{25}{s(s+4)}.$$

$\Rightarrow$ $\omega_n^2 = 25$

$$\boxed{\omega_n = 5 \text{ rad/sec}}$$

$\Rightarrow$ $\cancel{2}\xi\omega_n = \cancel{4}.2$

$\xi \times 5 = 2$

$$\boxed{\xi = 0.4}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 5 \times \sqrt{1-0.16}$$

$$\boxed{\omega_d = 4.58 \text{ rad/sec}}$$

$\Rightarrow$ $t_d = \dfrac{1 + 0.7\xi}{\omega_n}$

$$t_d = \frac{1 + (0.7 \times 0.4)}{5}$$

$$\boxed{t_d = 0.256}$$

$\Rightarrow$ $t_r = \dfrac{\pi - \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}}{\omega_d}.$

$$= \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

$$\therefore t_r = \frac{3.14 - \cos^{-1}(0.4)}{4.58}$$

$$\therefore \boxed{t_r = 0.4326 \text{ sec}}$$

$$\Rightarrow t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4.58}$$

$$\therefore \boxed{t_p = 0.686 \text{ sec}}$$

$$\Rightarrow M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\ \%$$

$$= e^{-3.14 \times 0.4/\sqrt{0.84}} \times 100\ \%$$

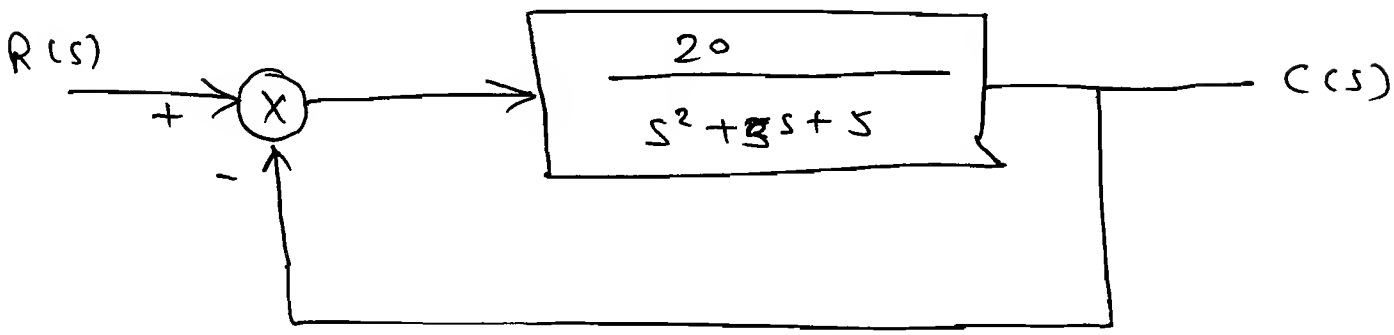
$$\boxed{M_p = 25.4\ \%}$$

$$\Rightarrow t_s = 4\tau = \frac{4}{\xi\omega_n}$$

$$t_s = \frac{4}{0.4 \times 5}$$

$$\therefore \boxed{t_s = 2 \text{ sec}}$$

Q Repeat the above problem,



Soln: $G(s) = \frac{20}{s^2 + 3s + 5}$.

$\Rightarrow$ ~~[illegible]~~

$$\frac{C(s)}{R(s)} = \frac{20}{s^2+5s+25}$$

$$\frac{C(s)}{R(s)} = \frac{20}{25}\left[\frac{25}{s^2+5s+25}\right].$$

→ This change effect the ss value but do not effect the T.D. specification.

$\Rightarrow \omega_n^2 = 25$

$$\boxed{\omega_n = 5 \text{ rad/sec}}$$

$2\xi\omega_n = 5$

$2 \times \xi \times \cancel{5} = \cancel{5}$

$$\boxed{\xi = 0.5}$$

$\Rightarrow \tau = \frac{1}{\xi\omega_n} = \frac{1}{5 \times 0.5}$

$$\boxed{\tau = 0.4 \text{ sec}}$$

$\omega_d = \omega_n\sqrt{1-\xi^2}.$

$= 5\sqrt{1-0.25}$

$$\boxed{\omega_d = 4.33 \text{ rad/sec}}$$

$\Rightarrow t_s = 4\tau$

$$\boxed{t_s = 1.6 \text{ sec}}$$

$\Rightarrow t_d = \frac{1+0.7\xi}{\omega_n}.$

$= \frac{1+(0.7\times0.5)}{5}$

$$\therefore \boxed{t_d = 0.27 \text{ sec}}$$

$\Rightarrow t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$

$t_r = \frac{\pi - \cos^{-1}(0.5)}{4.33}$

$$\therefore \boxed{t_r = 0.483 \text{ sec}}$$

$\Rightarrow$ $t_p = \dfrac{\pi}{\omega_d}$

$= \dfrac{3.14}{4.33}$

$\boxed{t_p = 0.725 \text{ sec}}$

$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$

$= e^{-3.14\times0.5/\sqrt{0.75}} \times 100\%$

$\boxed{M_p = 16.31\%}$

$\Rightarrow$ the unit step response to the above system is,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t + \cos^{-1}\xi).$$

$$\boxed{c(t) = 1 - \frac{e^{-2.5t}}{\sqrt{0.75}} \cdot \sin(4.33t + 60^\circ).}$$

$\Rightarrow$ Variation in time domain specification w.r.t. ξ (ω_n = constant).

$\Rightarrow$ As ξ increases from 0 to 1, the poles moves towards the left and near to the real axis.

$\Rightarrow$ In this case
1. $T \downarrow$
2. $t_s \downarrow$
3. $\omega_d \downarrow$

⇒ As ω_d decreases the time domain Specification (t_d, t_r & t_p) ↑.

⇒ As ξ increas from 0 to 1, % M_p decreases and the system become more Relatively Stable.

Q Find the T.D. Specification to the following system

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x.$$

Soln:

$$\frac{C(s)}{R(s)} = \frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}.$$

⇒ $\omega_n^2 = 8$

$\omega_n = 2\sqrt{2}$ rad/sec.

$\boxed{\omega_n = 2.83 \text{ rad/sec}}$

$\boxed{\xi = 0.707}$

$\cancel{2}\xi\omega_n = \cancel{4}\ 2$

$\xi = \frac{2}{2.83}$.

$\xi = 1/\sqrt{2} = 0.707$.

⇒ $\tau = \frac{1}{\xi\omega_n} = \frac{1}{\frac{1}{\sqrt{2}} \times 2\sqrt{2}} = \frac{1}{2\times 2} = 0.5$

$\boxed{\tau = 0.5 \text{ sec}}$

⇒ $t_s = 4\tau = 4\times 0.5$

$\boxed{t_s = 2 \text{ sec}}$

$$\Rightarrow \quad t_d = \frac{1 + 0.7\xi}{\omega_n}$$

$$= \frac{1 + (0.7 \times 0.707)}{2.83}$$

$$\therefore \boxed{t_d = 0.53 \text{ sec}}$$

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\boxed{\omega_d = 2 \text{ rad/sec}}$$

$$\therefore \quad t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

$$\therefore t_r = \frac{\pi - \cos^{-1} 0.707}{2}$$

$$\therefore \boxed{t_r = 1.177 \text{ sec}}$$

$$t_p = \pi / \omega_d$$

$$= 3.14 / 2$$

$$\boxed{t_p = 1.57 \text{ sec}}$$

$$M_p = e^{-\pi\xi / \sqrt{1-\xi^2}} \times 100 \%$$

$$= e^{-3.14 \times 0.707 / \sqrt{1-0.707^2}} \times 100 \%$$

$$\Rightarrow \boxed{M_p = 4.33 \%}$$

☆ Steady State errors:

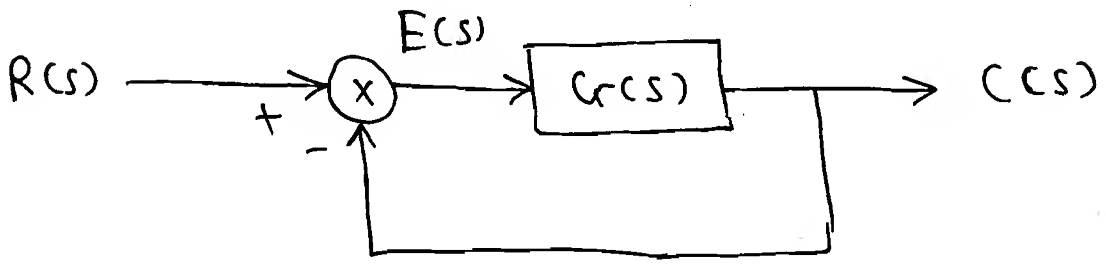
⇒ The error is nothing but deviation of the output from the input.

⇒ Steady state error is the error at $t \to \infty$.

$$e_{ss} = \lim_{t \to \infty} e(t).$$

$$\therefore \boxed{e_{ss} = \lim_{s \to 0} s\,E(s).}$$ ($\because$ Final value ~~theorm~~ theorem).

⇒ Consider the UFB system as shown in figure:



⇒ $E(s) = R(s) - C(s)$

$C(s) = G(s) \cdot E(s).$

$\therefore E(s) = R(s) - G(s) \cdot E(s).$

⇒ $$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)}.$$

$$\therefore \boxed{e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s)}} \leftarrow \text{H.B}$$

⇒ The Steady State errors depends on two factors ① Type of Input.

② Type of System.

⇒ The Steady State error are calculate to only CL Stable system.

⇒ The Steady State errors are Valid only for UFB System.

⇒ If Non Unity FB System is given it Should be Converted into UFB.

* **Type of Input:**

	Step	Ramp	Parabolic
I/P $r(t)$	$A\,u(t)$	$A t\,u(t)$	$A\,t^2/2\,u(t)$
e_{ss}	$\frac{A}{1+K_p}$	$\frac{A}{K_v}$	$\frac{A}{K_a}$
Error Constant	K_p: Possion cont $\lim_{s \to 0} G(s)$	K_v: velocity cons. $\lim_{s \to 0} s\,G(s)$.	K_a: Acceleration cons. $\lim_{s \to 0} s^2 G(s)$.

* Type of System:

⇒ The standard form of the system is

$$G(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s^{n}(1+sT_a)(1+sT_b)\cdots}$$

$H(s) = 1$

$\rightarrow$ Type $= n$ system.

⇒ Consider the step input and the different type of the system.

⊙ 0. Step ⇒ $(t^{0}.u(t))$.

⇒ $$e_{ss} = \frac{A}{1+K_p}$$

$$K_p = \lim_{s \to 0} G(s).$$

① Type - 0:

$$K_p = \lim_{s \to 0} \frac{K(1+sT_1)(1+sT_2)\cdots}{s^{0}(1+sT_a)(1+sT_b)\cdots}$$

$$\boxed{K_p = K}$$

⇒ $$e_{ss} = \cancel{K}\,\frac{A}{1+K_p} = \frac{A}{1+K}.$$

$$\boxed{e_{ss} = \frac{A}{1+K} = \text{constant}}$$

② Type - 1:

$\Rightarrow$ $$K_p = \lim_{s \to 0} \frac{K (1+ s\tau_1)(1+ s\tau_2)\cdots}{s^1 (1+ s\tau_a)(1+ s\tau_b)\cdots}$$

$\boxed{K_p = \infty.}$ $\Rightarrow$ $e_s = \frac{A}{1+\infty} = 0$

$\Rightarrow$ $\boxed{e_{ss} = 0}$

③ Type - 2:

$\Rightarrow$ $$K_p = \lim_{s \to 0} \frac{K (1+ s\tau_1)(1+ s\tau_2)\cdots}{s^2 (1+ s\tau_a)(1+ s\tau_b)\cdots}$$

$\boxed{K_p = \infty}$ $\Rightarrow$ $\boxed{e_{ss} = 0}$

$\Rightarrow$ The S.S. errors are required to calculate only in 3- cases:

i.e ① Type - 0 & step input (t^0).

② Type - 1 & ramp input (t^1).

③ Type - 2 & Parabolic input (t^2).

$\Rightarrow$ Remain all the cases the steady state error either become zero (or) infinity.

=>

Type = i/p	e_{ss} = Constant
Type > i/p	e_{ss} = 0.
Type < i/p	$e_{ss} = \infty$.

$K = \dfrac{\text{Nr. constant}}{\text{Dr. constant}}$

A = Amplitude of i/p.

$\boxed{Q}$ Find e_{ss} to the given unity FB system $G(s) = \dfrac{10(s+1)}{s^2(s+2)(s+10)}$, $H(s) = 1$.

to the following input

$$r(t) = (10 + 2t + 1 \cdot t^2/2) \cdot u(t).$$

Sol^n:

$$R(s) = \frac{10}{s} + \frac{2}{s^2} + \frac{1}{s^3}.$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)}.$$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s \cdot \left(\frac{10}{s} + \frac{2}{s^2} + \frac{1}{s^3} \right)}{1 + \frac{10(s+1)}{s^2(s+2)(s+10)}}.$$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{(10s^2 + 2s + 1)(s+2)(s+10)}{s^2(s+2)(s+10) + 10(s+1)}.$$

$$= \frac{(0+2)(0+10)}{0 + 10(0+1)} = \frac{20}{10} = 2.$$

<u>Method-2:</u> Compare type and input.

→ Type - 2:

Type		i/p		e_{ss}
2	>	0	→	0.
				+
2	>	1	→	0
				+
2	=	2	→	$\frac{A}{K}$

$$= \frac{\frac{1}{10 \times 1}}{10 \times 2}$$

$$+ \quad = 2$$

$$\boxed{e_{ss} = 2}$$

Q Find the e_{ss} to the following i/ps to the given UFB system.

$$G(s) = \frac{10}{s(s+5)} ; \quad H(s) = 1.$$

① $10u(t)$ ③ $10t^2u(t)$ ⑤ $(1+t+t^2)u(t)$.

② $10t\,u(t)$ ④ $(1+t)u(t)$

<u>Soln:</u> $G(s) = \frac{10}{s(s+5)}$.

<u>Type-1:</u>

① Type > (i/p = 0)

So, $\boxed{e_{ss} = 0}$

② Type = (i/p = 1)

$$\therefore \quad e_{ss} = \frac{\cancel{A}}{K} = \frac{1\cancel{0}5}{2\cancel{10}/\cancel{8}}$$

$$\boxed{e_{ss} = 5}$$

③ Type < (i/p = 2)

$$\therefore \quad \boxed{e_{ss} = \infty}$$

④ $(1+t)u(t) = u(t) + tu(t)$

$u(t) \downarrow e_{ss} = 0$

$tu(t) \downarrow e_{ss} = \frac{A}{K} = \frac{1}{10/5}$

$$\boxed{e_{ss} = 0.5}$$

⑤ $(1+t+t^2)u(t)$.

$= u(t) + tu(t) + t^2u(t)$

$$\downarrow \quad \boxed{e_{ss_1} = 0} + \boxed{e_{ss_2} = 0.5} + \boxed{e_{ss_3} = \infty}$$

$$\Downarrow$$

$$\boxed{e_{ss} = \infty}$$

[a] Repeat the above Problem for

$$G(s) = \frac{(s+1)}{s^2(s+5)(s+10)}.$$

Soln: Type - 2:

① Type > (i/p = 0)

$$\boxed{e_{ss} = 0}$$

② Type > (i/p = 1)

$\Rightarrow \boxed{e_{ss} = 0}$

③ Type = (i/p = 2)

$\therefore \quad e_{ss} = A/K = \dfrac{2 \times 10}{\frac{1}{5} \times 10} = 1000$

(∵ given is $t^2 u(t)$).

④ Type > i/p

$\Rightarrow \boxed{e_{ss} = 0}$

⑤ $(1 + t + t^2) u(t)$

$$= \underset{\substack{\downarrow \\ 0}}{u(t)} + \underset{\substack{\downarrow \\ 0}}{t \cdot u(t)} + \underset{\substack{\downarrow \\ e_{ss} = 100}}{t^2 u(t)}$$

$\therefore \quad \boxed{e_{ss} = 100}$

Q Repeat the above Problem for

$$G(s) = \frac{1}{s^2 (s+5)(s+10)}, \quad H(s) = 1.$$

Sol^n:

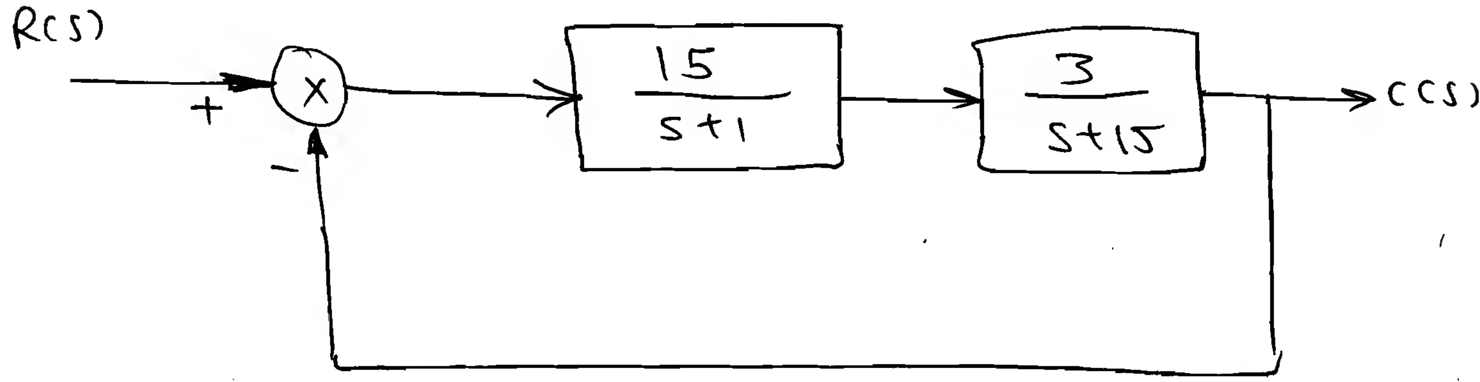
The above system is unstable,

$$CLTF = \frac{G}{1+G} = \frac{1}{s^4 + 15s^3 + \underbrace{50s^2 + 1}_{s^1 \text{ missing} \to US}}$$

$\Rightarrow$ The e_{ss} are calculated to only CLTF Stable system.

Note: → Before Calculating SS error observe the options. If any one of option is 'none' then verify the CLTF system stability by using the RH Criteria.

$\boxed{a}$ Calculate the e_{ss} to the given UFB system to the unit step input.



Sol^n: $G(s) = \dfrac{45}{(s+1)(s+15)}$.

$\Rightarrow$ (Type = 0) = (I/p = 0)

$\Rightarrow$ SSE $e_{ss} = \dfrac{A}{1+K} = \dfrac{1}{1+\frac{45}{15}}$

$\Rightarrow \boxed{e_{ss} = 0.25}$

Note: e_{ss} are calculated to only UFB system by using OLTF i.e. $G(s)$, $H(s) = 1$.

Method : 2 :

$\Rightarrow$ If any BO (or) SFG is given (or) Non UFB is given, then

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} s[R(s) - C(s)].$$

$$= \lim_{s \to 0} s.R(s) \left[1 - \frac{C(s)}{R(s)}\right].$$

$$\boxed{e_{ss} = \lim_{s \to 0} s.R(s)[1 - CLTF].}$$ Imp.

so, $\text{CLTF} = \dfrac{45}{s^2 + 16s + 60}$.

$$\therefore e_{ss} = \lim_{s \to 0} \cancel{s}.\left(\frac{1}{\cancel{s}}\right)\left(1 - \frac{45}{s^2+16s+60}\right).$$

$$= 1 - \frac{45}{60}$$

$$= 1 - 3/4.$$

$$= 1/4.$$

$$\Rightarrow \boxed{e_{ss} = 0.25}$$

$\boxed{Q}$ The OLTF of UFBS

$$G(s) = \frac{K}{s(s+1)(s+2)}$$. The value of

K to get the s.s error 0.1 to the unit ramp i/p is — ?

Sol^n:

Type = i/p

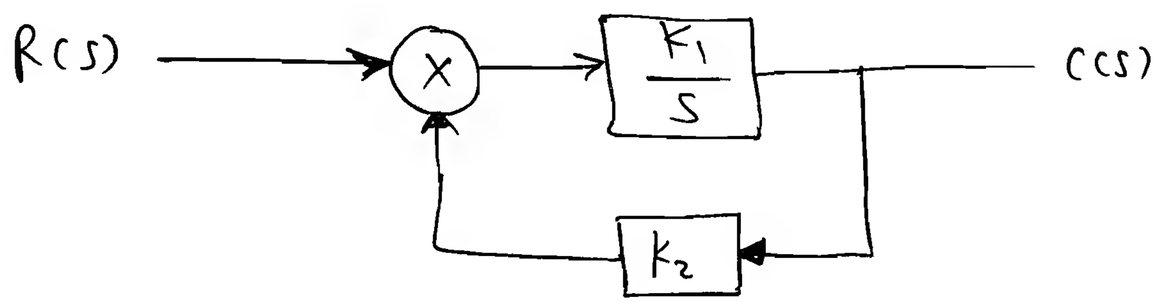
$$\Rightarrow \quad e_{ss} = \frac{A}{K}$$

$$\therefore \quad 0.1 = \frac{1}{K/(1\times2)}$$

$$\therefore \quad K = \frac{2}{0.1}.$$

$$\boxed{K = 20}$$

$\boxed{Q}$ For the system shown in fig. the s.s. o/p is 2. for unit step input and system time constant is 0.4 sec the values of K_1 & K_2 are ?



Sol^n:

$$CLTF = \frac{C(s)}{R(s)} = \frac{K_1/s}{1 + \frac{K_1 \cdot K_2}{s}} = \frac{K_1}{s + K_1 \cdot K_2}.$$

$$\therefore \quad \tau = \frac{1}{K_1 \cdot K_2} = 0.4.$$

Now, $\quad C_{ss} = C(t) = C(\infty) = 2.$

$$\therefore \quad C(\infty) = \lim_{t \to \infty} c(t)$$

$$= \lim_{s \to 0} s\, C(s).$$

$$C(\infty) = \lim_{s \to 0} s \cdot R(s) \cdot \frac{K_1}{s + K_1 \cdot K_2}$$

$$\therefore \quad 2 = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{K_1}{s + K_1 \cdot K_2}.$$

$$2 = \frac{\cancel{K_1}}{\cancel{K_1} \cdot K_2}$$

$$\boxed{K_2 = 0.5}$$

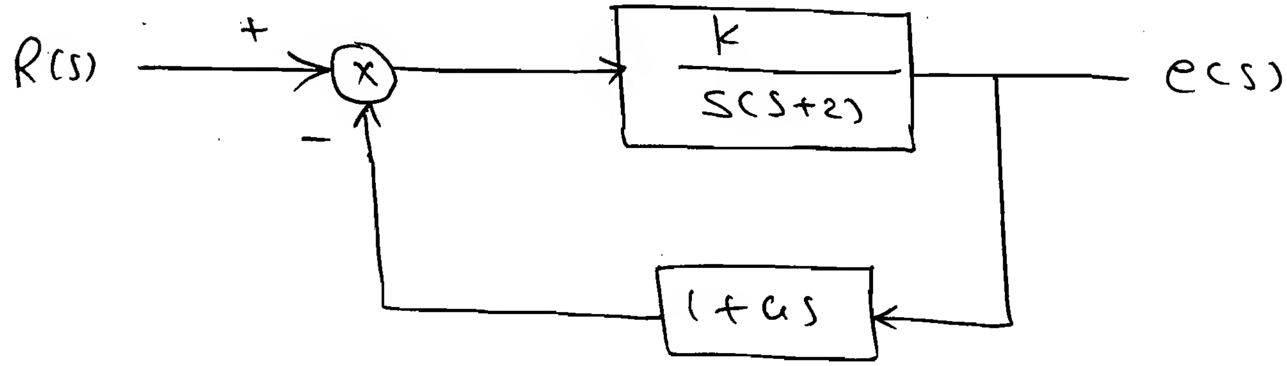
Now, $\quad K_1 \cdot K_2 = \dfrac{1}{0.4}$

$$K_1 = \frac{1}{0.4 \times 0.5}$$

$$\boxed{K_1 = 5}$$

$\boxed{a}$ For the system shown in figure the natural freq. of osc. is 4 rad/s and damping ratio is 0.7. The values of K & A are?

Soln: $\quad \omega_n = 4$ rad/sec., $\quad \zeta = 0.7$



$$\Rightarrow \quad CLTF = \frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+2)}}{1 + \frac{K(1+as)}{s(s+2)}}$$

$$= \frac{K}{s^2 + 2s + K + Kas}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{K}{s^2 + (2 + Ka)s + K}$$

$\Rightarrow \quad \omega_n^2 = K$

$\boxed{K = 16}$

$$2\xi\omega_n = 2 + Ka$$

$$\therefore 2 \times 0.7 \times 4 = 2 + Ka$$

$$3.6 = Ka$$

$$\Rightarrow a = \frac{3.6}{16}$$

$$\boxed{a = 0.225}$$

$\boxed{Q}$ A control system describe by following differential eqn.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 5y = 10(1 - e^{-2t}).$$

the response at $t \to \infty$.

Solⁿ: L.T.

$$\Rightarrow \quad (s^2 + 5s + 5)\,Y(s) = 10\left(\frac{1}{s} - \frac{1}{s+2}\right)X(s).$$

$$\therefore \quad \text{CLTF} \quad \frac{Y(s)}{X(s)} = \frac{\dfrac{20}{s(s+2)}}{(s^2+5s+5)}$$

$$\frac{Y(s)}{X(s)} = \frac{20}{s(s+2)(s^2+5s+5)}.$$

$$\therefore \quad y(\infty) = \lim_{t\to\infty} y(t) \qquad \boxed{X(s) = 1}$$

$$\therefore \quad Y(s) = \lim_{s\to 0} s \cdot Y(s).$$

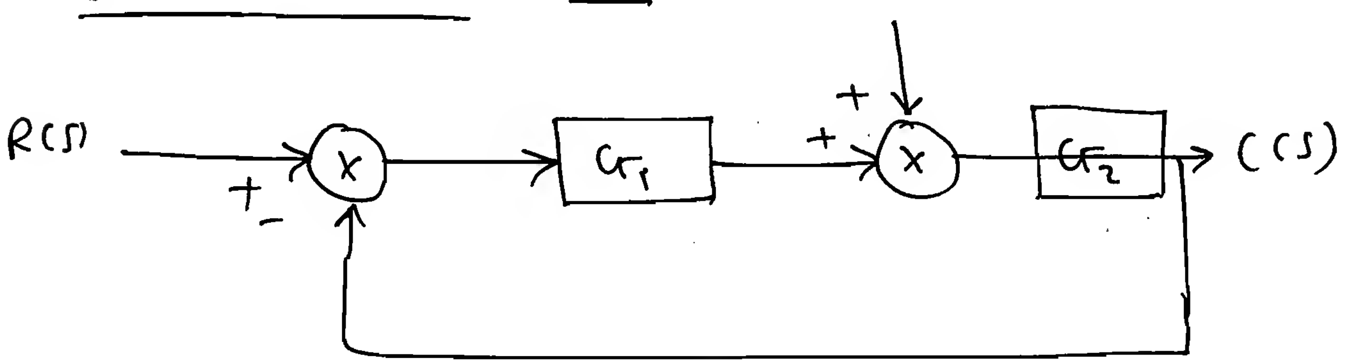
$$= \lim_{s\to 0} \cancel{s} \cdot \times \cancel{\tfrac{1}{s}} \times \frac{20}{\cancel{s}(s+2)\cdot(s^2+5s+5)}.$$

$$= \frac{20}{2 \times 5}$$

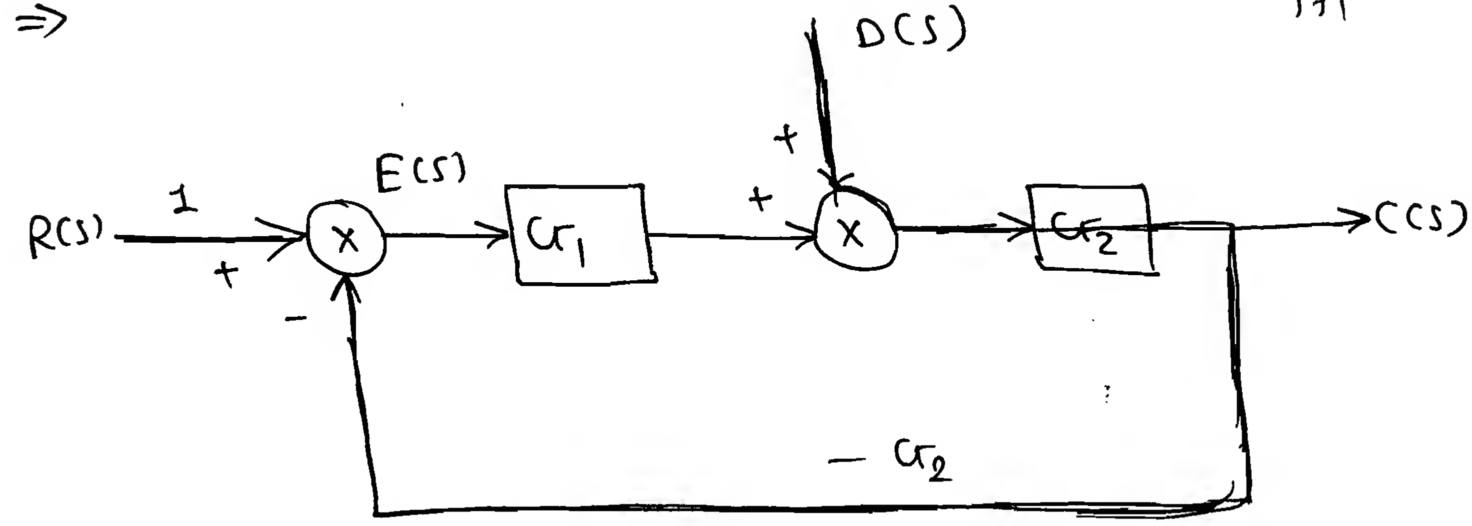
$$\therefore \quad \boxed{y(\infty) = 2}$$

* Steady State errors to the Disturbance i/p:

$\Rightarrow$



⇒



① e_{ss} due to R(s):

$$\frac{E(s)}{R(s)} = \frac{1}{1+G_1 \cdot G_2}.$$

$$\therefore \quad e_{ss_1} = \lim_{s \to 0} \frac{s \cdot R(s)}{1+G_1 \cdot G_2}.$$

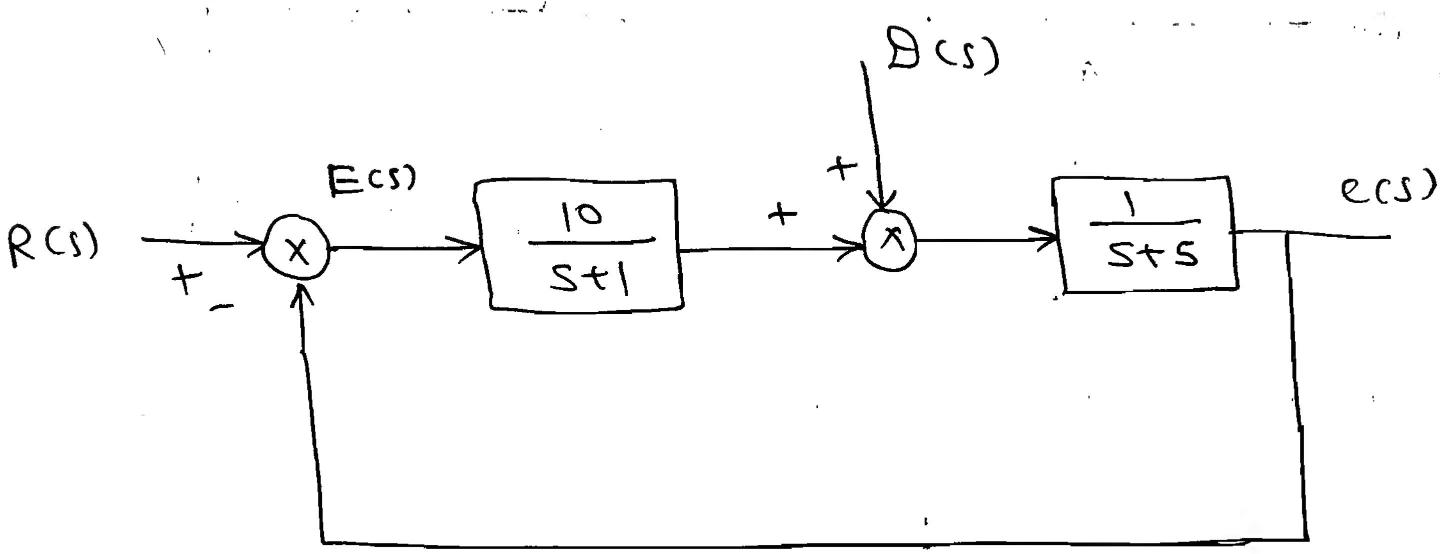
② e_{ss} due to D(s):

$$\therefore \quad \cancel{p} \quad \frac{E(s)}{D(s)} = \frac{-G_2}{1+G_1 \cdot G_2}.$$

$$\therefore \quad e_{ss_2} = \lim_{s \to 0} \frac{s \cdot [-G_2 \cdot D(s)]}{1+G_1 \cdot G_2}.$$

$$\therefore \quad \boxed{e_{ss} = \lim_{s \to 0} s \left[\frac{R(s) - G_2 \cdot D(s)}{1+G_1 \cdot G_2} \right].}$$

Q Find the e_{ss} due to the step input and step disturbance to the following system:



Soln:

$$e_{ss} = \lim_{s\to 0} s\left[\frac{R(s) - G_2(s).D(s)}{1 + G_1(s).G_2(s)}\right]$$

$$= \lim_{s\to 0} s\left[\frac{\frac{1}{s} - \frac{1}{s(s+5)}}{1 + \frac{10}{(s+1)(s+5)}}\right]$$

$$= \lim_{s\to 0}\left[\frac{1 - \frac{1}{s+5}}{1 + \frac{10}{(s+1)(s+5)}}\right]$$

$$= \frac{1 - 1/5}{1 + 10/1\times5}$$

$$\boxed{e_{ss} = \frac{4}{15}}$$

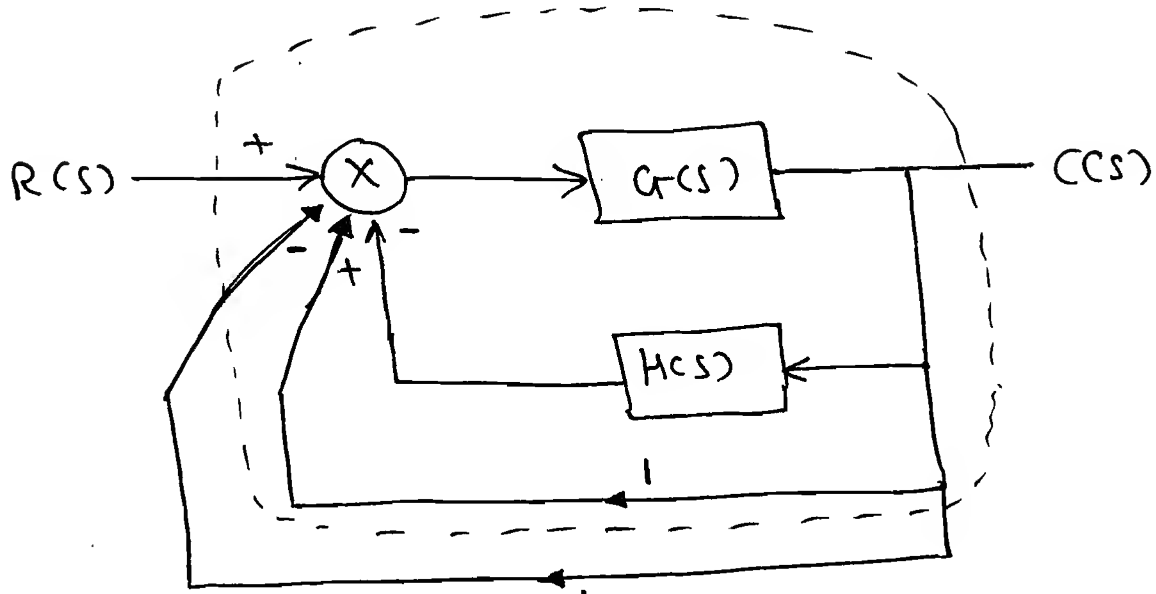
* Steady State error ~~due~~ to Non UFB System:

173

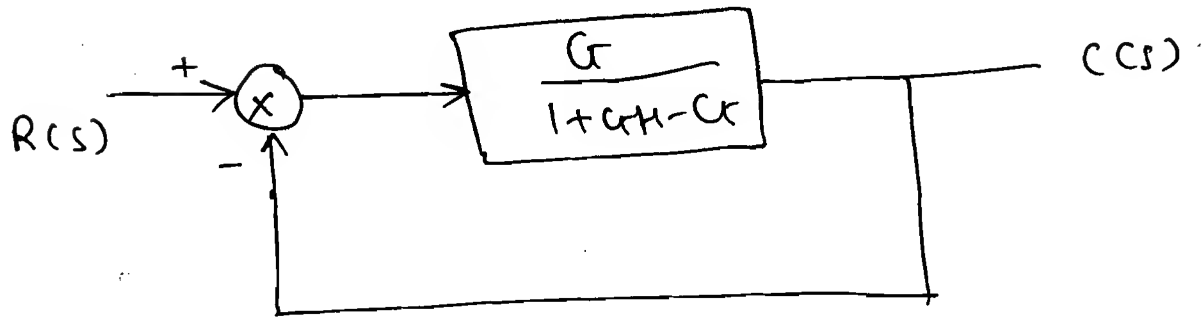
⇒ The ss errors are calculate to only a closed Loop Stable UFB System.

⇒ If Non-UFB system is given it should be converted into UFB as follows:

⇒



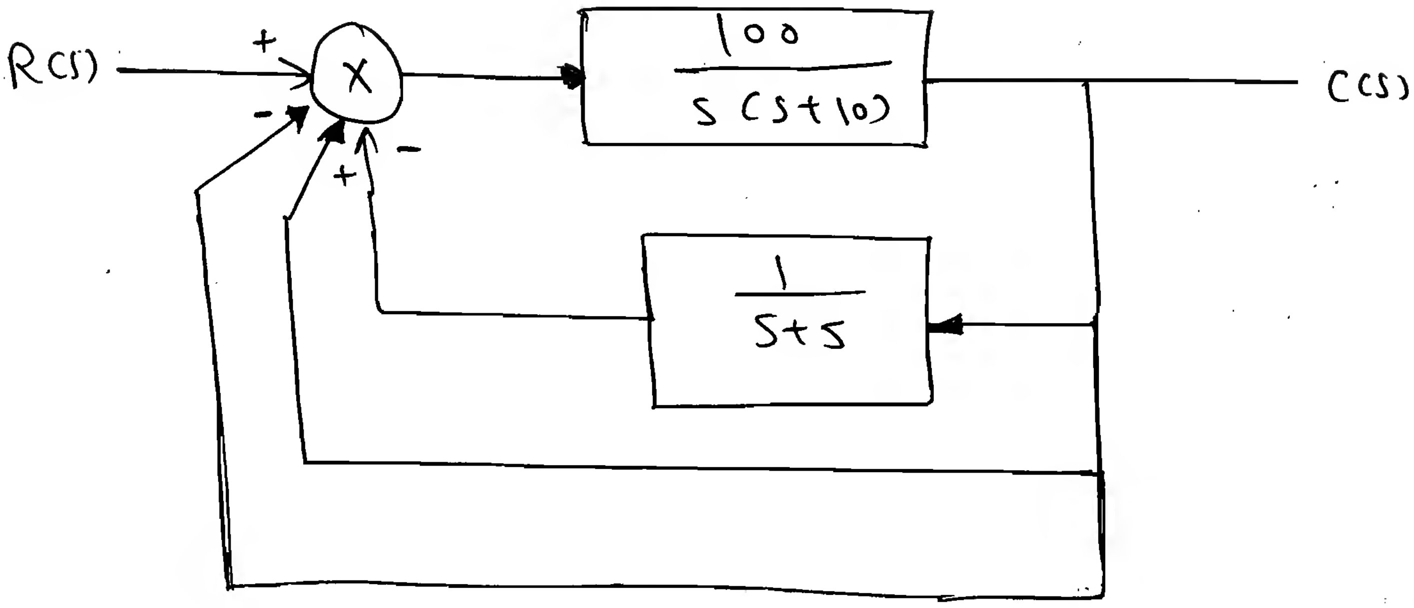
⇓



$$\Rightarrow \boxed{G(s) = \frac{G}{1 + GH - G}}$$

eqn OLTF of NUFB System.

Q Find the e_{ss} to the given NUFB system to unit step input.



Soln:

$$G_{NUF}(s) = \frac{G}{1 + GH - G}$$

$$= \frac{\frac{100}{s(s+10)}}{1 + \frac{100}{s(s+5)(s+100)} - \frac{100}{s(s+10)}}$$

$$G_{NUF}(s) = \frac{100(s+5)}{s(s+5)(s+100) + 100 - 100(s+5)}$$

$$= \frac{100s + 500}{s^3 + 105s^2 + 500s + 100 - 100s + 500}$$

$$G_{NUF}(s) = \frac{100s + 500}{s^3 + 105s^2 + 400s - 400}$$

(Type = 0) = (I/P = 0).

$$\therefore \quad e_{ss} = \frac{A}{1+K}$$

$$= \frac{1}{1 + \frac{500}{-400}} = \frac{1}{1-5/4} = -4$$

$$\therefore \quad \boxed{e_{ss} = -4}$$

Method - 2:

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \times \frac{1}{\cancel{s}}}{1 + \frac{100(s+5)}{s^3 + 105s^2 + 400s - 400}}$$

$$= \frac{1}{1 - \frac{500}{400}}$$

$$e_{ss} = \frac{+4}{4-5}$$

$$\Rightarrow \quad \boxed{e_{ss} = -4}$$